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Vortices in trapped boson-fermion mixtures

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We consider a trapped system of atomic boson-fermion mixture with a quantized vortex. We investigate the density profiles of bosonic and fermionic components as functions of the boson-boson and boson-fermion short-range interaction strengths within the mean-field approach. Stability of a vortex and conditions for the phase segregation are studied. We compare and contrast our results with the related system of droplets of ^{3}He-^{4}He mixtures.

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1. INTRODUCTION

After the successful achievement of Bose-Einstein condensation in dilute alkali gases[1] under magneto-optical trap potentials, a vast theoretical and experimental activity on cold degenerate quantum gases has followed.[2] More recently, fermionic gases are cooled to quantum degeneracy temperatures facilitated by mixing with cold bosonic gases by a process known as sympathetic cooling. Experimental progress in this direction has culminated in achieving the realization of quantum degenerate Bose-Fermi mixtures by several groups.[3–8] Currently there are a number of experiments on boson-fermion mixtures in harmonic traps. In the Paris experiment[4] ^{6}Li-^{7}Li mixture with a repulsive boson-fermion scattering length, and in the Florence experiment[8] ^{40}K-^{87}Rb mixture with an attractive boson-fermion scattering length are realized. Furthermore, using Feshbach resonances many groups have tuned the scattering length both for bosons and fermions. Theoretical studies on trapped boson-fermion mixtures employed the mean-field theory at zero temperature to determine the density profiles of respective components.[9–12] Related properties such as the stability against phase separation and collapse were also investigated.[13,14] The temperature effects and their role in phase separation were addressed by Akdeniz *et al.*[15] The critical temperature of the Bose-Einstein condensation in a trapped mixture were considered by several groups.[16,17]

Motivated by these recent experiments on boson-fermion mixtures of dilute alkali gases, in this paper, we study the ground state properties of such system in the presence of a single vortex. The quantized vortices are important in establishing the superfluid nature of Bose condensates.[2] Recently there has been numerous experimental works devoted to the creation and investigation of properties of quantized vortices in trapped condensates.[18] We are also motivated by the analogies and differences of trapped quantum gases and Helium droplets as prototypes of finite quantum fluids as recently surveyed by Dalfovo and Stringari.[19] To this end, we make contact with recent theoretical calculations of a vortex state in ^{3}He-^{4}He droplets.[20–22]

We employ the mean-field theory at zero temperature to consider a mixture of Bose condensed atoms and spin-polarized gas of fermions in a harmonic trap. Introducing a single quantized vortex through the Feynman-Onsager ansatz we study the ensuing density profiles of respective species. The density profiles are obtained by solving the mean-field equations for the trapped boson-fermion mixture using a variational ansatz.

2. MODEL AND THEORY

We consider N_B bosons of mass m_B in the condensed state and N_F fermions of mass m_F in respective trap potentials $V_B = \frac{1}{2}m_B\omega_B^2 r^2$ and $V_F = \frac{1}{2}m_F\omega_F^2 r^2$ in the form of isotropic harmonic oscillators. ω_B and ω_F are the trap frequencies for bosonic and fermionic species, respectively. The ground-state energy functional of a mixture of bosons and fermions in the mean-field approximation is given by

$$E[n_B(r), n_F(r)] = \int d\mathbf{r}\, (E_B + E_F + E_{BF})\,.$$

The energy density of bosons is

$$E_B = \frac{\hbar^2}{2m_B}|\nabla\Psi(r)|^2 + V_B(r)n_B(r) + \frac{g}{2}n_B(r)^2\,, \tag{1}$$

where $\Psi(r)$ is the condensate wavefunction, $n_B(r) = |\Psi(r)|^2$ is the condensate density distribution, and g is the boson-boson interaction strength. Since the fermions are assumed to be noninteracting, we have

$$E_F = T_F[n_F(r)] + V_F(r)n_F(r)\,, \tag{2}$$

where the kinetic energy functional for fermions with single spin species in the Thomas-Fermi approximation[9,10,14] is $T_F = (6\pi^2 n_F)^{5/3}/20\pi^2 m_F$ and the fermion density distribution is $n_F(r) = \frac{(2m)^{3/2}}{6\pi^2}[\varepsilon_F - V_F(r) - hn_B(r)]^{3/2}$ with ε_F the Fermi energy. The boson-fermion interaction energy density is $E_{BF} = hn_F(r)n_B(r)$, where h is the boson-fermion interaction strength. The total energy-density for the mixture now becomes

$$E[n_B, n_F] = \int d^3r \left[\frac{\hbar^2}{2m_B}|\nabla\Psi(r)|^2 + V_B(r)|\Psi(r)|^2 + \frac{g}{2}|\Psi(r)|^4 \right.$$
$$\left. T_F(n_F) + V_F(r)n_F(r) + hn_F(r)|\Psi(r)|^2 \right] . \qquad (3)$$

We have assumed that the fermionic component of the mixture is spin-polarized whereby the $s-$wave scattering between the fermions is inhibited by the Pauli principle. g and h are the boson-boson and boson-fermion interaction strengths, respectively, related to the $s-$wave scattering lengths a_{BB} and a_{BF} as measured in experiments,[4,5] viz. $g = 4\pi\hbar^2 a_{BB}/m_B$ and $h = 4\pi\hbar^2 a_{BF}/\mu_{BF}$, where μ_{BF} is the reduced mass. We introduce a quantized vortex through the Feynman-Onsager ansatz, $\Psi(r) = \psi(r)e^{i\phi}$, which amounts to adding a centrifugal energy term $\frac{\hbar^2}{2m_B r^2}|\psi|^2$ to the total energy functional. Our goal is to minimize the total energy functional subject to the normalization conditions $\int d\mathbf{r}\, n_B(r) = N_B$ and $\int d\mathbf{r}\, n_F(r) = N_F$.

To study the density profiles of boson and fermion components of the mixture, we now introduce the variational wavefunction for the condensate with a vortex, $\Psi(r,\phi) = Are^{i\phi}e^{-\alpha r^2}$ where A is the normalization constant and α is the variational parameter. The normalization integral for N_B bosons yields $A = N_B^{1/2}(128\alpha^5/9\pi^3)$.

3. RESULTS AND DISCUSSION

We have minimized the total energy of the mixture with respect to the variational parameter α and the fermion density $n_F(r)$ using the number of particles N_B and N_F as constraints. We have assumed the same mass $m_B = m_F$ for both species and the same trap frequency $\omega_B = \omega_F$ for simplicity. The boson-boson and boson-fermion interaction strengths are treated as tunable parameters.

In Fig. 1 we show the density profile of fermion species $n_F(r)$ as a function of the radial coordinate for various values of the repulsive boson-fermion interaction strength. For fixed boson-boson repulsive interaction ($g = 0.005\hbar\omega a_{HO}^3$ in the examples shown) we observe a depletion in the central region of the fermion density as boson-fermion interaction strength increases. The dip in the central region of $n_F(r)$ coincides with the maximum of the density profile of the condensate with a vortex. Further increase in h

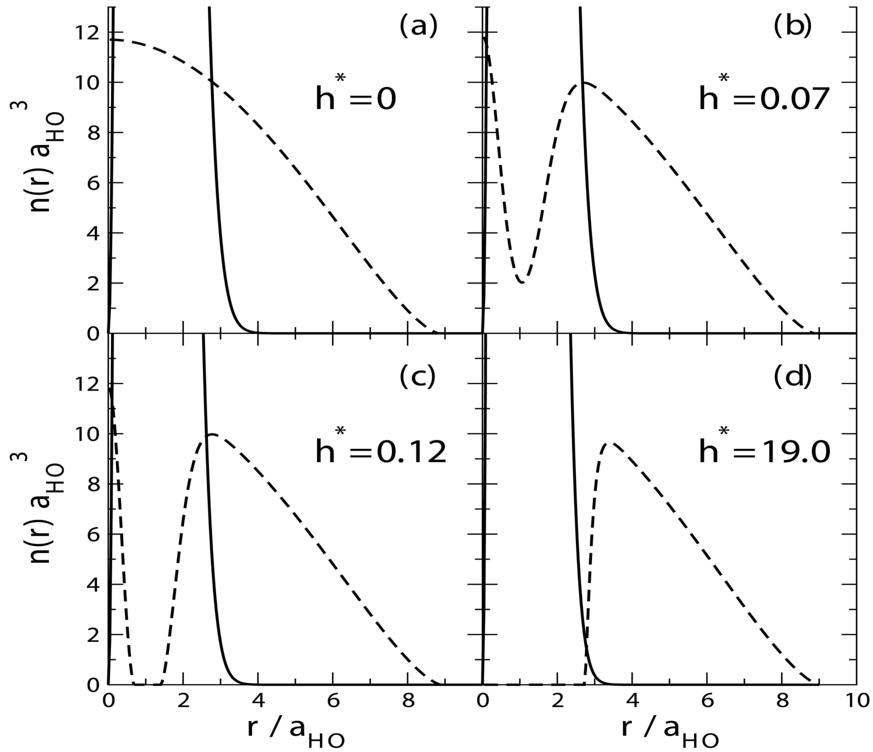

Fig. 1. Density profile of fermions $n_F(r)$ (dashed lines) for $g = 0.005\hbar\omega\, a_{HO}^3$ and $N_B = N_F = 10^4$ particles. $h^* = h/(\hbar\omega\, a_{HO}^3)$. Solid lines indicate the boson density $n_B(r)$.

causes the break up of fermion density into two parts, one filling the vortex core, the other part pushed to the outer region [Fig. 1(c)]. Eventually, when h becomes very large, the fermions disappear from the vortex core region and occupy only the outer region surrounding the condensate [Fig. 1(d)]. This last situation is the phase separated case of two species, similar to the theoretically calculated case of trapped boson-fermion mixtures without a vortex.[6,7] The small overlap of boson $n_B(r)$ and fermion $n_F(r)$ densities is an artifact of Gaussian variational wavefunction which would give in to a complete phase segregation in more elaborate calculations.

We point out the similarity between our results shown in Fig. 1 and those of Mayol *et* al.[20] who considered quantized vortices in ^{3}He-^{4}He droplets. They have found that even a small number of ^{3}He atoms fills the vortex

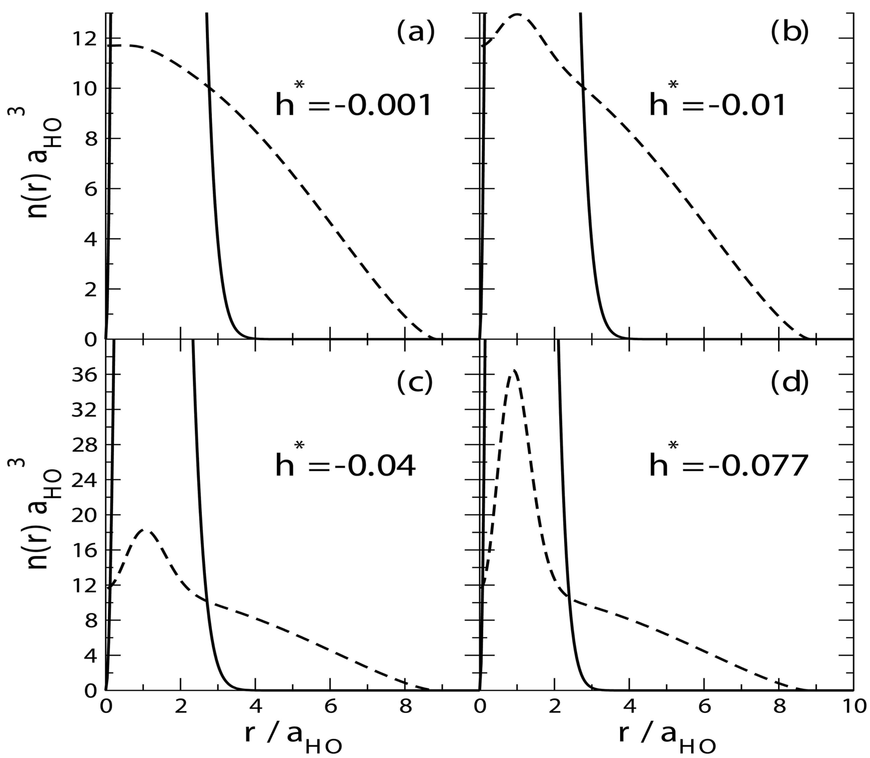

Fig. 2. Density profile of fermions $n_F(r)$ (dashed lines) for $g = 0.005\hbar\omega\, a_{HO}^3$ and $N_B = N_F = 10^4$ particles. $h^* = h/(\hbar\omega\, a_{HO}^3)$. Solid lines indicate the boson density $n_B(r)$.

core provided by the quantized vortex in a ^{4}He condensate. Whereas in the case of ^{3}He-^{4}He mixtures the strong interaction potential between He atoms is fixed, the interactions between the alkali atoms can be tuned by Feshbach resonances to study a wider range of density profiles and possible phase separations.

We next consider attractive interactions between bosons and fermions. As shown in Fig. 2 an attractive boson-fermion interaction strength causes the central region of the fermion density $n_F(r)$ to increase. At a critical value of h the system becomes unstable and the fermionic component collapses much like the situation in vortex-free boson-fermion mixtures studied previously.[7,8]

Our variational calculations employing a Gaussian ansatz may be improved by choosing better variational wavefunctions or numerically solving the coupled Euler-Lagrange equations for the mixture. We surmise, however, the results reported here should be qualitatively correct.

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REFERENCES

1. M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995); K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
2. F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999); A. J. Leggett, *ibid.* **73**, 307 (2001).
3. S. G. Crane, X. Zhao, W. Taylor, and D. J. Vieira, Phys. Rev. A **62**, 011402 (2000).
4. F. Schreck *et* al. Phys. Rev. Lett. **87**, 080403 (2001).
5. J. Goldwin, S. B. Papp, B. DeMarco, and D. S. Jin, Phys. Rev. A **65**, 021402 (2002).
6. Z. Hadzibabic *et* al. Phys. Rev. Lett. **88**, 160401 (2002).
7. G. Ferrari *et* al. Phys. Rev. Lett. **89**, 053202 (2002).
8. G. Roati *et* al. Phys. Rev. Lett. **89**, 150403 (2002).
9. K. Mølmer, Phys. Rev. Lett. **80**, 1804 (1998).
10. N. Nygaard and K. Mølmer, Phys. Rev. A **59**, 2974 (1999).
11. T. Miyakawa, T. Suzuki, and H. Yabu, Phys. Rev. A **64**, 033611 (2001); T. Miyakawa, K. Oda, T. Suzuki, and H. Yabu, J. Phys. Soc. Jpn. **69**, 2779 (2000).
12. M. Amoruso, A. Minguzzi, S. Stringari, M. P. Tosi, and L. Vichi, Eur. Phys. J. D **4**, 261 (1998).
13. R. Roth and H. Feldmeier, Phys. Rev. A **65**, 021603 (2002).
14. R. Roth, Phys. Rev. A **66**, 013614 (2002).
15. Z. Akdeniz *et* al. J. Phys. B **35**, L105 (2002); Phys. Rev. A **66**, 013620 (2002).
16. Y.-L. Ma and S.-T. Chui, Phys. Rev. A **66**, 053611 (2002).
17. A. P. Albus *et* al. J. Phys. B **35**, L551 (2002).
18. K. W. Madison *et* al. Phys. Rev. Lett. **84**, 806 (2000); B. P. Anderson *et* al. *ibid.* **85**, 2857 (2000); J. R. Abo-Shaeer *et* al. Science **292**, 476 (2001).
19. F. Dalfovo and S. Stringari, J. Chem. Phys. **115**, 10 078 (2001).
20. R. Mayol, M. Pi, and M. Barranco, Phys. Rev. Lett. **87**, 145301 (2001).
21. F. Dalfovo, R. Mayol, M. Pi, and M. Barranco, Phys. Rev. Lett. **85**, 1028 (2000).
22. H. Shi and S. T. Chui, Physica B **284-288**, 166 (2003).