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Joint decisions on inventory replenishment and emission reduction investment under different emission regulations

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Carbon emission regulation policies have emerged as mechanisms to control firms' carbon emissions. To meet regulatory requirements, firms can make changes in their production planning decisions or invest in green technologies. In this study, we analyse a retailer's joint decisions on inventory replenishment and carbon emission reduction investment under three carbon emission regulation policies. Particularly, we extend the economic order quantity model to consider carbon emissions reduction investment availability under carbon cap, tax and cap-and-trade policies. We analytically show that carbon emission reduction investment opportunities, additional to reducing emissions as per regulations, further reduce carbon emissions while reducing costs. We also provide an analytical comparison between various investment opportunities and compare different carbon emission regulation policies in terms of costs and emissions. We document the results of a numerical study to further illustrate the effects of investment availability and regulation parameters.

Keywords: green technology; carbon emissions; investment; economic order quantity

1. Introduction and literature review

Global warming, environmental disasters and increased public awareness about environmental issues are encouraging countries to reduce greenhouse gas (GHG) emissions. The Kyoto Protocol, signed in 1997 by 37 industrialised countries and European Union (EU) members, enabled nations to aggregately focus on GHG emission abatement. Several government programs (e.g. the EU Emissions Trading System, the New Zealand Emissions Trading Scheme, the US' Regional Greenhouse Gas Initiative), private voluntary-membership organisations (e.g. the Chicago Climate Exchange, the Montreal Climate Exchange) and many emissions-offset companies have emerged as control mechanisms over firms' GHG emissions, primarily carbon emissions (other GHG emissions can be measured in terms of equivalent carbon emissions, see, e.g. EPA 2013). To reduce carbon emissions, policy-makers either provide incentives to achieve emission reduction or impose costs on carbon emissions.

Under carbon emission regulation policies, firms seek cost-efficient methods to decrease emissions, mainly through replanning (changing) their operations and investing in carbon emission abatement (Bouchery et al. 2011). A firm can reduce its carbon emissions level via changing its production, inventory, warehousing, logistics and transportation operations (Benjaafar, Li, and Daskin 2013; Hua, Cheng, and Wang 2011). For instance, after 60,000 suppliers of Wal-Mart decreased their packaging by 5% upon Wal-Mart's request, they achieved 667,000 m³ of CO₂ emission reduction (Hoffman 2007). Hewlett-Packard (HP) reported that they decreased toxic inventory release to the air from 26.1 tonnes to 18.3 tonnes in 2010 by adjusting operations (HP 2011).

A firm can also reduce its carbon emissions level by directly investing in carbon emission reduction projects such as greener transportation fleets (see, e.g. Bae, Sarkis, and Yoo 2011), energy-efficient warehousing (see, e.g. Ilic, Staaake, and Fleisch 2009) and environmentally friendly manufacturing processes (see, e.g. Liu, Anderson, and Cruz 2012). McKinsey & Company reports that US carbon emissions can be reduced by three to 4.5 gigatons in 2030 using tested approaches and high-potential technologies (Creyts et al. 2007). Additional to directly investing in carbon emission reduction projects that decrease emissions from internal operations, companies can indirectly invest in carbon emission reduction by purchasing carbon offsets (see, e.g. Benjaafar, Li, and Daskin 2013; Song and Leng 2012), which can compensate for a company's carbon emissions and be used to increase its carbon emissions cap. Carbon-offset projects are referred to as clean development mechanisms (CDM) under the Kyoto Protocol. The United Nations Framework for Convention on Climate Change provides a list of

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CDM; see, e.g., <http://cdm.unfccc.int/>. The World Bank reports that the global carbon market, including traded allowances and offset transactions, reached \$176 billion in 2011 (Kossoy and Guigon 2012).

Common carbon emission regulations include cap, cap-and-trade and tax policies. Under the cap policy, a firm's carbon emissions should not exceed a predetermined amount, which is referred to as a carbon cap. The cap can be determined by a government agency and/or the firm's green goals (Chen, Benjaafar, and Elomri 2013). Under the cap-and-trade policy, carbon emissions are tradable through a system such as the EU Emissions Trading System or the New Zealand Emissions Trading Scheme; a firm can buy or sell carbon allowances at a specified market price. Under the tax policy, a firm is charged for its carbon emissions through taxes. In this paper, we study a retailer's joint decisions on inventory replenishment and carbon emission reduction investment under these three policies.

As the world economy becomes increasingly conscious of the environmental concerns, evidence suggests that companies who make better business decisions to consider the interests of other stakeholders, including the human and natural environments, will succeed (Jaber 2009). While the environmental regulation policies aim to protect consumers, employees and the environment, cost of compliance should not deter companies to do business. Inventories play an important role in the operations and the profitability of a company. Therefore, one of our goals in this paper is to provide guidance to the companies to make better inventory decisions while utilising the available environmental technologies under different regulation policies. Our other purpose is to help policy-makers understand the implications of each regulation policy on the profitability of a company, and the role that green technologies play in the resulting carbon emissions and costs of the company.

Most papers focusing on replanning inventory replenishment decisions for environmental considerations, study the classic economic order quantity (EOQ) setting. Hua, Cheng, and Wang (2011) analyse the EOQ model under the cap-and-trade policy. They investigate how replenishment decisions, costs and carbon emissions change with the market price of carbon trading. Chen, Benjaafar, and Elomri (2013) study the EOQ model with the cap policy and examine its effects on carbon emissions and costs. They also discuss the applicability of their results under tax, cap-and-offset and cap-and-price policies. Our study is similar to Hua, Cheng, and Wang (2011) and Chen, Benjaafar, and Elomri (2013) in that we also take the perspective of a retailer operating in the EOQ environment and consider the existence of a carbon regulation policy. However, ours is more general due to the fact that we study the retailer's investment decisions along with his/her replenishment decisions. It should be noted that there are also studies that propose extensions of the EOQ model with environmental considerations in the absence of carbon emission regulation policies. For instance, Bonney and Jaber (2011) introduce costs into the EOQ model associated with carbon emissions and disposed wastes due to transportation and inventory operations. Bouchery et al. (2012) formulate a multi-objective EOQ model that minimises costs and environmental damages. It is worthwhile noting that along with the ordering decisions in the EOQ setting, the product-mix problem (Letmathe and Balakrishnan 2005), dynamic economic lot sizing problem (Absi et al. 2013; Benjaafar, Li, and Daskin 2013), single-period stochastic replenishment problem (Song and Leng 2012), transport mode selection (Hoen et al. 2013) and two-echelon production planning (Jaber, Glock, and El Saadany 2013; Saadany, Jaber, and Bonney 2011) are among the issues that have been revisited in regard to environmental considerations.

As noted, leading companies in their sectors invest to decrease the environmental effects of their products and production and logistical processes, or to curb emissions through offset projects. Although investment decisions for environmental considerations is still a developing area in the operations research and management science literature, it is possible to classify the related studies in three groups. The first group of papers study the ordering and investment decisions in settings where consumer demand is sensitive to the environmental quality of the product, which in turn, can be increased through investment (e.g. Swami and Shah 2013; Zavanella et al. 2013). Note that these studies do not consider any regulation policies; the only motivation for investing in greening efforts is to increase demand by improving customers' perception of the product. The second group of papers model carbon offset investments when a cap-and-offset policy is in place (e.g. Benjaafar, Li, and Daskin 2013; Chen, Benjaafar, and Elomri 2013; Song and Leng 2012). A cap-and-offset policy can be considered as a mix of cap and cap-and-trade policies. It differs from a cap policy in that the carbon allowance can be increased with offset investments. It differs from a cap-and-trade policy in that it does not allow carbon allowances to be tradable. The second group of studies exhibit two important characteristics. First, all three papers (i.e. Benjaafar, Li, and Daskin 2013; Chen, Benjaafar, and Elomri 2013; Song and Leng 2012) assume unit reduction in carbon emissions per unit investment (which is included as an additional component in the cost function). Second, this type of investment modelling (i.e. offset investments) is not relevant within the context of other regulation policies. The final group of studies consider investing in technology to reduce emissions under a regulation policy. We have identified only one paper that falls into this group, i.e. Jiang and Klabjan (2013), taking a firm's perspective to analyse the effects of investment decisions on the profitability and carbon emissions. Our paper also contributes to the third group of literature by modelling and solving a retailer's joint inventory replenishment and carbon emission reduction investment decisions under each of the three stated carbon emission regulation policies. Examples of investment opportunities for emission reduction include purchasing more efficient

electric appliances (lights, refrigerators, etc.); improving the energy efficiency of existing appliances and equipment related to lighting, air-conditioning, water heating; fuel switching; deriving energy through renewable energy sources.

Jiang and Klabjan (2013) analyse production and carbon emission reduction investment decisions under different regulation policies (i.e. cap-and-trade, command-and-control). They consider a setting in which carbon trading price and demand are stochastic, and assume a linear investment function. The decision-maker first decides on production capacity and carbon emission reduction investment, and then, after the carbon trading price and demand are realised, the operations are adjusted. The authors extend this model to analyse investment timing decisions in two periods. Our paper differs from Jiang and Klabjan (2013) in two major ways. First, we analyse the classic EOQ model with an investment option under cap, tax and cap-and-trade policies. Second, we consider a non-linear investment function. We treat the investment amount as capital expenditure, similar to Billington (1987), that is, some amount of money is invested per unit time and the reduction in carbon emissions per unit time is a function of the invested money. We benefit from Huang and Rust (2011) in creating a correlation between investment and carbon emission reduction. Huang and Rust (2011) note that spending on green technologies has decreasing marginal returns in pollution/environmental damage reduction. Therefore, the firm's carbon emission reduction per unit time is assumed to be an increasing concave function of the investment money per unit time. Through this functional form, we generalise the linear relation (i.e. constant marginal returns of the investment amount in carbon emission reduction) assumed by Benjaafar, Li, and Daskin (2013), Chen, Benjaafar, and Elomri (2013), Jiang and Klabjan (2013), and Song and Leng (2012).

We provide a solution method for a retailer's joint inventory control and carbon emission reduction investment decisions for each carbon regulation policy considered. The resulting optimal values of the order quantity and the yearly investment amount under a certain policy simultaneously minimise the retailer's average annual costs if that policy is in place. Following this analysis, we compare the retailer's annual costs and carbon emissions with and without investment availability under each carbon regulation policy. We analytically show that availability of carbon emission reduction investment, additional to the reductions achieved by carbon emission regulation policies, further reduces carbon emissions while reducing costs under the tax and cap-and-trade policies. Under the cap policy, the resulting emissions level does not decrease due to investment; however, the same emissions level is achieved with lower costs. Therefore, we conclude that it is more important for governments to stimulate green technology under the tax and cap-and-trade policies. Several investment options with varying cost and carbon emission reduction characteristics may be available to the retailer. The retailer may thus need to select one investment opportunity. We provide analytical and numerical comparisons of the resulting costs and carbon emissions between different investment opportunities available to the retailer under each carbon emission regulation policy.

Our analysis enables comparing carbon emission regulation policies with the carbon emission reduction investment option. Our results indicate that when the retailer can invest in carbon emission reduction, compared to a given tax policy, a cap policy that will lower costs and not increase carbon emissions is possible. Furthermore, we show that for any given cap policy, there exists a cap-and-trade policy that will lower costs and carbon emissions. Further analytical and numerical results are discussed about the effects of policy parameters on the retailer's costs and emissions. These results can be utilised by policy-makers in legislating carbon emissions or in constructing specific carbon emission regulation policies.

The rest of the paper is organised as follows: In Section 2, we describe the setting and the problem in more detail. Section 3 presents solutions for the retailer's order quantity and carbon emission reduction investment decisions under cap, tax and cap-and-trade policies. In this section, we also present the analytical results on the benefits of the carbon emission reduction investment option and the comparison of different carbon emission reduction investment opportunities. We compare the carbon regulation policies in Section 4 and summarise our numerical studies in Section 5. We conclude the paper with some final remarks in Section 6.

2. Problem definition

In this study, a retailer's emission reduction investment and inventory replenishment decisions are analysed under different government regulations on carbon emissions. It is assumed that the retailer operates under the conditions of the classical EOQ model. That is, the retailer orders Q units at each replenishment to meet deterministic and steady demand on time in the infinite horizon. In the setting of interest, there is significant carbon emission due to ordering, inventory holding and procurement. The carbon emitted per replenishment, per-unit purchase and per-unit per-year inventory holding amount to \hat{A} , \hat{c} and \hat{h} , respectively.

We consider three different carbon emission policies: cap, tax and cap-and-trade. Under the cap policy, the retailer's carbon emissions per year cannot exceed an emission cap, denoted by C . Under the tax policy, the retailer is taxed p monetary units for unit carbon emission. Under the cap-and-trade policy, the retailer can trade a unit carbon emission for a value of c_p monetary units. These policies are intended to reduce carbon emissions by affecting the retailer's operations; however, the retailer can also reduce his/her carbon emissions by investing in new technology, equipment or machinery.

Table 1. Problem parameters and decision variables.

Retailer's parameters	
A	Fixed cost of inventory replenishment
h	Cost of holding one unit inventory for a year
c	Unit procurement cost
D	Demand per year
\hat{A}	Carbon emission amount due to inventory replenishment
\hat{h}	Carbon emission amount due to holding one unit inventory for a year
\hat{c}	Carbon emission amount due to unit procurement
Policy parameters	
i	Carbon policy index; $i = 1$ for cap, $i = 2$ for tax, and $i = 3$ for cap-and-trade policies
C	Annual carbon emission cap
p	Tax paid for one unit of emission
c_p	Unit carbon emission trading price
Retailer's decision variables	
Q	Order quantity
G	Annual investment amount for carbon emission reduction
X	Traded quantity of emission capacity in cap-and-trade policy
Functions and optimal values of decision variables	
$TC(Q, G)$	Total average annual costs as a function of Q and G without a carbon policy
$E(Q, G)$	Carbon emissions per year as a function of Q and G
$TC_i(Q, G)$	Total average annual costs as a function of Q and G under carbon policy i
Q_i^*	Optimal order quantity under carbon policy i
G_i^*	Optimal investment amount under carbon policy i

Mainly, annual carbon emission can be decreased in an amount of $\alpha G - \beta G^2$ in return for G monetary units invested per year ($0 \leq G \leq \frac{\alpha}{\beta}$). Here, α reflects the efficiency of green technology in reducing emissions, and β is a decreasing return parameter (Huang and Rust 2011). In each case, the problem is to find the order quantity and the investment amount that jointly minimise the retailer's total average annual costs. Table 1 summarises the notation used in the paper. Additional notation will be defined as needed.

Without any carbon emission policy in place, the total average annual costs due to ordering, inventory holding, procurement and investment is given by

$$TC(Q, G) = \frac{AD}{Q} + \frac{hQ}{2} + cD + G, \quad (1)$$

and the total average annual emission amount is given by

$$E(Q, G) = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2. \quad (2)$$

When the retailer makes no investment, i.e. $G = 0$, Expression (1) provides the total average annual costs in the EOQ model, and its value is minimised at $Q^0 = \sqrt{\frac{2AD}{h}}$, which we refer to as the 'cost-optimal quantity'. If there is no carbon emission policy in place, $(Q^0, 0)$ will in fact be the optimising pair of order quantity and investment amount for the retailer. Furthermore, it follows from Expression (2) that $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ is the minimum average annual carbon emission possible without investment, and is achieved when the retailer orders $Q^e = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$ units, which we refer to as the 'emission-optimal quantity'.

The problem parameters are assumed to satisfy the following conditions:

- (A1) The minimum annual carbon emission possible due to ordering decisions is more than the maximum yearly emission reduction possible due to investment decisions. That is,

$$\sqrt{2\hat{A}\hat{h}D} + \hat{c}D > \frac{\alpha^2}{4\beta}. \quad (3)$$

(A2) For the tax policy under consideration, there exists a value of $G > 0$ at which savings in taxes when G monetary units are invested in new technology to reduce carbon emissions exceeds the cost of investment. Hence, we have

$$\alpha p > 1. \quad (4)$$

(A3) For the cap-and-trade policy under consideration, there exists a value of investment amount $G > 0$ at which more reduction in carbon emissions can be achieved by investing in new technology rather than purchasing carbon capacity at a total value of G monetary units. Hence, we have

$$\alpha c_p > 1. \quad (5)$$

(A4) For the cap policy under consideration, there exist values of the investment amount that can reduce the annual carbon emission to below carbon capacity. Hence, we have

$$\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C. \quad (6)$$

The right-hand side of Inequality (3), that is, $\frac{\alpha^2}{4\beta}$, is the maximum possible value of annual carbon emission reduction and is achieved when $G = \frac{\alpha}{2\beta}$. Recall that $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ is the minimum possible value of yearly carbon emissions due to ordering decisions. An implication of Assumption (A1), therefore, is that carbon emissions cannot be completely eliminated with new technology. Assumption (A2), in mathematical terms, is equivalent to saying that there exists some $G > 0$ at which $(\alpha G - \beta G^2)p > G$. Dividing both sides of this inequality by G and considering the fact that $\beta G p > 0$ leads to $\alpha p > 1$. If Assumption (A2) does not hold, then any investment to reduce carbon emissions does not pay off, and hence, an investment decision should not be of concern. Similarly, Assumption (A3) can be written as $\alpha G - \beta G^2 > \frac{G}{c_p}$ for some positive value of G , which in turn implies $\alpha c_p > 1$. Finally, Assumption (A4) is necessary for the retailer to be in business under the current cap policy. If the minimum carbon emission possible (i.e. $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$) due to ordering and investment decisions were more than the cap C , then there would be no feasible solution to the retailer's inventory problem.

3. Analysis under different carbon emission policies

In this section, we solve the retailer's integrated problem of finding the optimal order quantity and carbon emission reduction investment under the three carbon emission regulation policies: cap, tax and cap-and-trade. We represent the optimal solution under each policy i as a pair of values (Q_i^*, G_i^*) . The proofs of all our results are presented in the Appendix.

Recall that, by definition of the investment function, there exists an upper bound on G , that is, $G \leq \frac{\alpha}{2\beta}$. We do not include this restriction as a constraint because the nature of our formulations for all emission regulations makes it redundant. That is, the investment value in all optimal solutions without incorporating $G \leq \frac{\alpha}{2\beta}$ already satisfies this constraint. In fact, due to the strict concavity of $\alpha G - \beta G^2$ with respect to G and the fact that $\frac{\alpha}{2\beta}$ is its unique maximiser, for every investment value that is greater than $\frac{\alpha}{2\beta}$, the corresponding reduction in annual carbon emission can be achieved by a smaller investment amount within the range $0 \leq G \leq \frac{\alpha}{2\beta}$. Therefore, the optimal investment value will always be less than or equal to $\frac{\alpha}{2\beta}$. The optimal solutions for the cap, tax and cap-and-trade policies, as they are stated in Theorems 1–3, justify these observations.

3.1 Cap policy

Under a cap policy, the retailer is subject to an upper bound, that is an 'emission cap', on the total average annual carbon emission. The retailer's problem is to find the optimal order quantity and the investment amount to minimise average annual total cost without exceeding the emission cap C . This problem can be formulated as follows:

$$\begin{aligned} \min TC_1(Q, G) &= \frac{AD}{Q} + \frac{hQ}{2} + cD + G \\ \text{s.t.} \quad &\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 \leq C, \\ &Q \geq 0, G \geq 0. \end{aligned}$$

Note that, when $G = 0$, there exists a feasible solution to the above problem as long as $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. Given that $G = 0$, the feasible region consists of all pairs $(Q, 0)$ such that $Q_1 \geq Q \geq Q_2$, where

$$Q_1 = \frac{C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \tag{7}$$

and

$$Q_2 = \frac{C - \hat{c}D - \sqrt{(C - \hat{c}D)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}. \tag{8}$$

Q_1 and Q_2 are the two roots of $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D = C$. It is important to note that the existence of Q_1 and Q_2 depend on how $(C - \hat{c}D)$ compares to $\sqrt{2\hat{A}\hat{h}D}$, and is not guaranteed. In fact, in Theorem 1, we characterise the optimal solution to the retailer’s problem in two parts, considering the following two cases: (i) $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and (ii) $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. In the latter case, the restriction on the maximum carbon emission cannot be overcome only by ordering decisions, the retailer must also take advantage of investment opportunities. Assumption (A4) guarantees that there exists a feasible solution in this case. Prior to stating the retailer’s optimal order quantity and investment decisions under a cap policy, let us also introduce the following solution pairs:

$$\begin{aligned} (Q_3, G_3) &= \left(\frac{(C - \hat{c}D + \alpha G_3 - \beta G_3^2) + \sqrt{(C - \hat{c}D + \alpha G_3 - \beta G_3^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \frac{2D(A\alpha + \hat{A}) - Q_3^2(\alpha h + \hat{h})}{2\beta(2AD - Q_3^2 h)} \right), \\ (Q_4, G_4) &= \left(\frac{(C - \hat{c}D + \alpha G_4 - \beta G_4^2) - \sqrt{(C - \hat{c}D + \alpha G_4 - \beta G_4^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \frac{2D(A\alpha + \hat{A}) - Q_4^2(\alpha h + \hat{h})}{2\beta(2AD - Q_4^2 h)} \right), \\ (Q_5, G_5) &= \left(Q^e, \frac{\alpha - \sqrt{\alpha^2 - 4\beta(-C + \hat{c}D + \sqrt{2\hat{A}\hat{h}D})}}{2\beta} \right). \end{aligned}$$

Note that $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 = C$ when (Q, G) is any one of the pairs (Q_3, G_3) , (Q_4, G_4) , and (Q_5, G_5) . For $0 \leq G \leq \frac{\alpha^e}{2\beta}$, it can be shown that

$$Q_3 \geq Q_1 \geq Q_2 \geq Q_4. \tag{9}$$

As characterised in the next theorem and its proof, the optimal solution to the retailer’s problem under the cap policy is given by one of the following pairs: $(Q^0, 0)$, $(Q_1, 0)$, $(Q_2, 0)$, (Q_3, G_3) , (Q_4, G_4) , and (Q_5, G_5) . If $(Q_1^*, G_1^*) = (Q^0, 0)$, then the cost-optimal solution satisfies the emission constraint already. If $(Q_1^*, G_1^*) = (Q_1, 0)$ or $(Q_1^*, G_1^*) = (Q_2, 0)$, then the retailer is able to satisfy the emission constraint by ordering a quantity other than the cost-optimal one while not making any investment. In other cases where $G_1^* > 0$, the retailer minimises his/her costs under the emission constraint by investing in new technology besides carefully made ordering decisions.

THEOREM 1 *Under a cap policy, the optimal pair of the retailer’s replenishment quantity and his/her investment amount is as follows:*

If $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ then,

$$(Q_1^*, G_1^*) = \begin{cases} (Q^0, 0) & \text{if } Q_2 \leq Q^0 \leq Q_1, \\ (Q_1, 0) & \text{if } Q^\alpha < Q_1 < Q^0, \\ (Q_3, G_3) & \text{if } Q^e < Q_3 \leq Q^\alpha, \\ (Q_2, 0) & \text{if } Q^0 < Q_2 < Q^\alpha, \\ (Q_4, G_4) & \text{if } Q^\alpha \leq Q_4 < Q^e, \end{cases}$$

and if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, then

$$(Q_1^*, G_1^*) = \begin{cases} (Q_3, G_3) & \text{if } Q^e < Q_3 \leq Q^\alpha, \\ (Q_4, G_4) & \text{if } Q^\alpha \leq Q_4 < Q^e, \\ (Q_5, G_5) & \text{o.w.,} \end{cases}$$

$$\text{where } Q^\alpha = \sqrt{\frac{2(\hat{A} + A\alpha)D}{\hat{h} + h\alpha}}.$$

The result that will be highlighted next, applies to the special case of the problem where $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$, and is a consequence of Theorem 1 and its proof.

Remark 1 If $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$, the optimal replenishment quantity is always given by the cost-optimal solution Q^0 , which is equal to the emission-optimal solution Q^e . However, if $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, then $G_1^* = 0$, and if $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, then $G_1^* > 0$.

It is worthwhile to note that, when there is no investment opportunity for carbon emissions reduction, Theorem 1 coincides with the results of Chen, Benjaafar, and Elomri (2013). The next corollary presents the annual carbon emission level resulting from the retailer's optimal decisions as given in Theorem 1.

COROLLARY 1 *The average annual carbon emission resulting from the retailer's optimal solution under a cap policy is*

$$E(Q_1^*, G_1^*) = \begin{cases} \frac{\sqrt{D(\hat{A}h + \hat{h}A)}}{\sqrt{2A\hat{h}}} + \hat{c}D & \text{if } Q_2 \leq Q^0 \leq Q_1, \\ C & \text{o.w.} \end{cases}$$

As seen in Corollary 1, the maximum carbon emissions per year are bounded by C . However, as long as C is not binding such that $Q_2 \leq Q^0 \leq Q_1$, annual carbon emissions are linearly increasing with \hat{A} and \hat{h} . For those nonbinding C values, annual carbon emissions are also dependent on an A/h ratio, and in fact, increases with A/h if $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. Furthermore, the carbon emissions level is not dependent on investment parameters α and β .

In the next lemma, we investigate the impact of having an investment option for carbon emission reduction on the retailer's annual emission level under a cap policy. In doing this, we consider the following two measures: $E(Q_1^*(0), 0) - E(Q_1^*, G_1^*)$ and $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*)$. We use the notation $Q_1^*(0)$ to refer to the retailer's optimal replenishment quantity under a cap policy, given that the investment amount is zero. Note that, a feasible value for $Q_1^*(0)$ may not always exist, specifically when $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. The lemma, which will be presented without a proof, follows from Corollary 1 and the expression for $E(Q_1^*(0), 0)$ provided in Chen, Benjaafar, and Elomri (2013). The result applies to cases in which a feasible value of $Q_1^*(0)$ can be found.

LEMMA 1 *Having an investment opportunity for carbon emission reduction does not change the annual carbon emission level under a cap policy, however, it may lead to lower average annual costs for the retailer. That is, $E(Q_1^*(0), 0) - E(Q_1^*, G_1^*) = 0$ and $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*) \geq 0$.*

If $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and an investment option is not available for the retailer to reduce his/her carbon emissions, there is no feasible replenishment quantity, and therefore it does not make sense for him/her to be in business. Therefore, in such cases, the savings in costs due to having an investment option may as well be considered as infinity. Note that when $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, $Q_1^*(0)$ is given by Q^0 if $Q_2 \leq Q^0 \leq Q_1$, by Q_2 if $Q^0 < Q_2$, and by Q_1 if $Q_1 < Q^0$. The optimal (Q, G) pairs in the problems with and without the investment option coincide in those cases. Therefore, the savings in costs due to investment can be strictly positive only under the circumstances in which $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, and the solution to the problem with investment option is given by either (Q_3, G_3) or (Q_4, G_4) .

Next, we study the effects of a cap policy on the retailer's annual carbon emissions and costs in comparison to a case where there is no governmental regulation. In the latter case, the retailer orders Q^0 units and makes no investment for emission reduction.

LEMMA 2 *Under a cap policy, the retailer's optimal decisions for replenishment quantity and investment amount may reduce the yearly carbon emissions with an annual cost that is no less than what it would be when no emission policy is in place. That is, $TC_1(Q_1^*, G_1^*) \geq TC(Q^0, 0)$ and $E(Q_1^*, G_1^*) \leq E(Q^0, 0)$.*

Under any of the emission regulation policies, there may exist investment options with different parameters α and β . If this is the case, then the retailer must choose among different investment options. The result presented in the next lemma may help the retailer to make such a decision when a cap policy is in place.

LEMMA 3 *Let us consider two feasible investment options (i.e. they satisfy Assumption (A4)): one with parameters α_1 and β_1 , and the other with parameters α_2 and β_2 . If $\beta_2 \geq \beta_1$ and $\alpha_2 \leq \alpha_1$, then under the first investment option, there exists a solution which leads to the same annual emission level with no more costs.*

The above lemma implies that between two different investment options, the retailer should choose the one with higher α and smaller β . If the investment option with higher α does not also have smaller β , we will show, in the numerical analysis in Section 5, that the problem parameters determine which investment option is better in terms of costs. Recall from Corollary 1 that the annual carbon emissions level under the cap policy is independent of the investment parameters α and β . Therefore, annual costs due to each investment option is the only criterion that determines which investment option is better.

3.2 Tax policy

Under a tax policy, the retailer pays p monetary units in taxes for unit carbon emission. There is no restriction on the maximum carbon emissions. The retailer's problem can be formulated as follows:

$$\begin{aligned} \min TC_2(Q, G) &= \frac{AD}{Q} + \frac{hQ}{2} + cD + G + pE(Q, G) \\ \text{s.t. } E(Q, G) &= \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2, \\ Q &\geq 0, G \geq 0. \end{aligned}$$

The following theorem characterises the solution to the above problem:

THEOREM 2 *Under a tax policy, the optimal pair of retailer's replenishment quantity and his/her investment amount is given by*

$$(Q_2^*, G_2^*) = \left(\sqrt{\frac{2(A + \hat{A}p)D}{h + \hat{h}p}}, \frac{\alpha p - 1}{2p\beta} \right).$$

It can be observed that G_2^* is increasing with p . Furthermore, Q_2^* is increasing with p when $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, Q_2^* is decreasing with p when $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, and Q_2^* is not affected by p when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. In fact, when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_2^* = Q^0 = Q^e$. The next corollary, which will be presented without a proof, follows from plugging the expressions for Q_2^* and G_2^* in the emission function and the cost function.

COROLLARY 2 *The average annual carbon emission and the average annual cost resulting from the retailer's optimal solution under a tax policy are*

$$E(Q_2^*, G_2^*) = \frac{\sqrt{D} [\hat{A}(h + p\hat{h}) + \hat{h}(A + p\hat{A})]}{\sqrt{2(A + p\hat{A})(h + p\hat{h})}} + \frac{1 - \alpha^2 p^2}{4p^2\beta} + \hat{c}D, \quad (10)$$

$$TC_2(Q_2^*, G_2^*) = \sqrt{2(A + p\hat{A})(h + p\hat{h})}D + D(c + \hat{c}p) - \frac{(\alpha p - 1)^2}{4p\beta}. \quad (11)$$

It can be verified by Assumptions (A1) and (A3) that $E(Q_2^*, G_2^*)$ and $TC_2(Q_2^*, G_2^*)$ are positive. $E(Q_2^*, G_2^*)$ is decreasing in p and $TC_2(Q_2^*, G_2^*)$ is increasing in p . In the next lemma, we quantify the reduction in emissions and the savings in costs due to the investment option. For this purpose, we consider the following two measures: $E(Q_2^*(0), 0) - E(Q_2^*, G_2^*)$ and $TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*)$. Here, $Q_2^*(0)$ refers to the retailer's optimal replenishment quantity under the tax policy, given that the investment amount is zero.

LEMMA 4 *Under a tax policy, having an investment opportunity for carbon emission reduction leads to positive savings in annual carbon emissions and in annual costs, as quantified by the following:*

$$E(Q_2^*(0), 0) - E(Q_2^*, G_2^*) = \frac{\alpha^2 p^2 - 1}{4p^2\beta}, \quad TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*) = \frac{(\alpha p - 1)^2}{4p\beta}.$$

Lemma 4 along with Assumption (A2) imply that the reduction in annual costs and the reduction in annual carbon emissions due to utilising the investment opportunity are both increasing in p . The reduction in annual carbon emissions is bounded by $\frac{\alpha^2}{4\beta}$ and its rate of change with increasing p decreases. This, in turn, implies that if the government further increases the tax for one unit of emission at its already large values, a retailer investing in new technology does very little to reduce emissions. However, the retailer still invests in new technology because he/she can reduce his/her costs significantly

by means of tax savings. Note that the total taxes the retailer must pay may be very large at high values of p , therefore, even a marginal reduction in emissions may save the retailer a lot of money.

In the next lemma, we study the effects of the carbon tax policy on the retailer's annual carbon emissions and costs. Without a carbon emission policy in place, the retailer minimises Expression (1), and he/she orders Q^0 units and makes no investment in emissions reduction.

LEMMA 5 *Under a tax policy, the retailer's cost-optimal decisions for replenishment quantity and investment amount lead to lower annual emissions and higher annual costs, in comparison to a case with no emission policy. That is, $TC_2(Q_2^*, G_2^*) > TC(Q^0, 0)$ and $E(Q_2^*, G_2^*) < E(Q^0, 0)$.*

The above lemma implies that a tax policy is effective in reducing a retailer's annual carbon emissions, but it increases the retailer's annual costs even if he/she has access to an investment opportunity for carbon emission reduction. In what follows, we compare two investment opportunities under the tax policy.

LEMMA 6 *Let us consider two investment options: one with parameters α_1 and β_1 , and the other with parameters α_2 and β_2 . When a tax policy is in place, the retailer's annual costs and emissions under one option compare to those under another in the following way:*

- If $\beta_2 \geq \beta_1$ and $\alpha_2 \leq \alpha_1$, then the first investment option (i.e. the one with parameters α_1 and β_1) leads to no greater annual emissions and no greater annual costs for the retailer than the second investment option does.
- If $\beta_2 \geq \beta_1$ and $\alpha_2 > \alpha_1$, then
 - If the second investment option leads to greater annual costs than the first one does, then it also results in greater annual emissions.
 - If the second investment option leads to annual costs lower than or equal to the first one, then it results in lower annual emissions if $\frac{1-\alpha_2^2 p^2}{\beta_2} < \frac{1-\alpha_1^2 p^2}{\beta_1}$ holds, otherwise, it results in no lower annual emissions than the first investment option does.

3.3 Cap-and-trade policy

Under a cap-and-trade policy, similar to the cap policy, the retailer is subject to an emissions cap, C , on the total carbon emissions per year. However, if the annual carbon emission is more than the cap C , the firm can buy carbon permits equivalent to its excess demand for carbon capacity, at a market price of c_p monetary units per unit emission. On the other hand, if the retailer's annual carbon emission is lower than the carbon cap, she/he can sell the extra carbon capacity at the same market price, i.e. c_p . It is assumed that carbon permits are always available for buying and selling. In particular, let X denote the carbon amount the retailer trades annually. $X > 0$ indicates a case in which the retailer sells his/her carbon permits, whereas $X < 0$ implies a case in which the retailer purchases carbon permits. The retailer's problem of deciding the replenishment quantity and the investment amount is formulated below.

$$\begin{aligned} \min \quad & TC_3(Q, G) = \frac{AD}{Q} + \frac{hQ}{2} + cD + G - Xc_p \\ \text{s.t.} \quad & \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 + X = C, \\ & Q \geq 0, G \geq 0. \end{aligned}$$

In the following theorem, we present the solution to the above problem:

THEOREM 3 *Under a cap-and-trade policy, the optimal pair of retailer's replenishment quantity and his/her investment amount is given by*

$$(Q_3^*, G_3^*) = \left(\sqrt{\frac{2(A + \hat{A}c_p)D}{h + \hat{h}c_p}}, \frac{\alpha c_p - 1}{2c_p \beta} \right).$$

It then follows that $X^* = C - E(Q_3^*, G_3^*)$, where X^* is the retailer's optimal traded carbon amount per year.

Using the expression for G_3^* , one can show that G_3^* is increasing with c_p . Furthermore, Q_3^* is increasing with c_p when $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, Q_3^* is decreasing with c_p when $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, and it is not affected by c_p when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. In case $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_3^* = Q^0 = Q^e$. The next three corollaries follow from Theorem 3.

COROLLARY 3 If $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} > C$, then the retailer does not sell any carbon permits (i.e. $X \leq 0$), regardless of what the carbon trading price c_p is.

At high values of c_p , the retailer may want to sell his/her permits in the market for extra revenue. However, Corollary 3 implies that if the cap is smaller than the minimum carbon emissions possible due to ordering and investment decisions, the retailer must purchase carbon permits to be within the allowed limits of annual carbon emissions at any value of c_p .

COROLLARY 4 The average annual carbon emissions and the average annual costs resulting from the retailer's optimal decisions under a cap-and-trade policy are

$$E(Q_3^*, G_3^*) = \frac{\sqrt{D}(\hat{A}(h + c_p\hat{h}) + \hat{h}(A + c_p\hat{A}))}{\sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})}} + \frac{1 - \alpha^2 c_p^2}{4c_p^2\beta} + \hat{c}D, \quad (12)$$

$$TC_3(Q_3^*, G_3^*) = \sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})D} + D(c + \hat{c}c_p) - \frac{(\alpha c_p - 1)^2}{4c_p\beta} - c_p C. \quad (13)$$

Equation (12) implies that the carbon emissions level does not change with carbon cap C . Hua, Cheng, and Wang (2011) obtain a similar result for the case when there is no investment option. It can be shown using Assumption (A3) that $E(Q_3^*, G_3^*) > 0$; however, $TC_3(Q_3^*, G_3^*)$ may assume any value depending on the magnitude of C . If $TC_3(Q_3^*, G_3^*) < 0$, then the retailer has excess carbon capacity in such a large amount that by selling this amount he/she covers the inventory-related costs and even makes a profit. (In practice, this should be avoided for the cap-and-trade policy to be effective.) Based on this result, the next corollary proposes an upper bound on the value of C that the policy-maker should impose on the retailer in this setting.

COROLLARY 5 Under a cap-and-trade policy with a carbon trading price c_p , an upper bound on the carbon capacity C is given by

$$C < \frac{\sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})D} + D(c + \hat{c}c_p) - \frac{(\alpha c_p - 1)^2}{4c_p\beta}}{c_p}.$$

To quantify the reduction in emissions and the savings in costs due to the investment option under a cap-and-trade policy, in the next lemma we consider the following two measures: $E(Q_3^*(0), 0) - E(Q_3^*, G_3^*)$ and $TC_3(Q_3^*(0), 0) - TC_3(Q_3^*, G_3^*)$. Here, $Q_3^*(0)$ refers to the retailer's optimal replenishment quantity under the cap-and-trade policy, given that the investment amount is zero.

LEMMA 7 Under a cap-and-trade policy, having an investment opportunity for carbon emission reduction leads to positive savings in annual carbon emissions and in annual costs, as quantified by the following:

$$E(Q_3^*(0), 0) - E(Q_3^*, G_3^*) = \frac{\alpha^2 c_p^2 - 1}{4c_p^2\beta}, \quad TC_3(Q_3^*(0), 0) - TC_3(Q_3^*, G_3^*) = \frac{(\alpha c_p - 1)^2}{4c_p\beta}.$$

Lemma 7 and Assumption (A3) jointly imply that the reduction in annual costs and the reduction in annual carbon emissions due to utilising the investment opportunity are both increasing in c_p . The reduction in annual carbon emissions is again bounded by $\frac{\alpha^2}{4\beta}$, as in the case of the tax policy, and, its rate of change with increasing c_p decreases. With an interpretation similar to the one we developed for Lemma 4, it can be concluded that the incremental benefit of retailer's one-unit investment on emission reduction diminishes at large values of unit carbon emission trading prices. However, the retailer still invests in new technology, because he/she can reduce his/her costs significantly either by creating excess carbon capacity and selling it at high prices, or by avoiding the need to purchase excess capacity at high prices with the capacity generated from new technology.

In the next lemma, we study the effects of the cap-and-trade policy on the retailer's annual carbon emissions and costs. For this purpose, we compare the annual carbon emissions and the annual costs in case of no government regulation to the results in Corollary 4. Note that, in the former case, the retailer orders Q^0 units and makes no investment in emission reduction.

LEMMA 8 Under a cap-and-trade policy, the retailer's cost-optimal decisions for replenishment quantity and investment amount lead to lower annual emissions in comparison to a case with no emission policy. That is, $E(Q_3^*, G_3^*) < E(Q^0, 0)$. However, annual costs may increase or decrease depending on C . Specifically, we have $TC_3(Q_3^*, G_3^*) \leq TC(Q^0, 0)$ if

$$C \geq \frac{\sqrt{2(A + \hat{A}c_p)(h + \hat{h}c_p)D} - \sqrt{2\hat{A}hD}}{c_p} - \frac{(\alpha c_p - 1)^2}{4c_p^2\beta} + \hat{c}D, \text{ and we have } TC_3(Q_3^*, G_3^*) > TC(Q^0, 0) \text{ otherwise.}$$

The next lemma, which will be presented without a proof, presents a result for the cap-and-trade policy, similar to the one in Lemma 6 for the tax policy.

LEMMA 9 *Let us consider two investment options: one with parameters α_1 and β_1 , and the other with parameters α_2 and β_2 . The retailer's annual costs and emissions under one option compare to those under the other in the following way:*

- *If $\beta_2 \geq \beta_1$ and $\alpha_2 \leq \alpha_1$, then the first investment option (i.e. the one with parameters α_1 and β_1) leads to no greater annual emissions and no greater annual costs for the retailer than the second investment option does.*
- *If $\beta_2 \geq \beta_1$ and $\alpha_2 > \alpha_1$, then*
 - *If the second investment option leads to greater annual costs than the first one does, then it also results in greater annual emissions.*
 - *If the second investment option leads to annual costs lower than or equal to the first one, then it results in lower annual emissions if $\frac{1-\alpha_2^2 c_p^2}{\beta_2} < \frac{1-\alpha_1^2 c_p^2}{\beta_1}$ holds, otherwise, it results in no lower annual emissions than the first investment option does.*

4. Analytical results on the comparison of the three emission policies

In Section 3, we derived analytical solutions to the retailer's problem of finding the replenishment quantity and the investment amount under the three carbon regulation policies. We obtained two sets of results: one about the impact of an investment opportunity on the annual costs and emissions (see Lemmas 1, 4 and 7), and the other about how the different emission policies change the retailer's annual costs and emissions in comparison to a no-policy case (see Lemmas 2, 5 and 8). Looking into the first set of results, we arrive at the following conclusions:

- Under any of the three carbon regulation policies, total annual costs without the investment option are greater than or equal to the total annual costs with the investment option.
- While annual carbon emissions levels with and without the investment option are equal under the cap policy, carbon emissions level without the investment option is greater than the carbon emissions level with the investment option under the tax policy and cap-and-trade policy.

The above results imply that having an investment option under a cap policy does not reduce the retailer's emission level in comparison to a case with no such option; however, it may help him/her achieve the same carbon amount with lower costs. On the other hand, having an investment option under a tax policy or a cap-and-trade policy has a more pronounced effect on the retailer's annual carbon emissions and costs: the retailer can take advantage of the investment option and reduce both his/her emissions and costs. From an environmental point of view, the above implies that an investment option along with a tax policy or a cap-and-trade policy as an emission regulation further enhances emission reduction. Therefore, governments should enable opportunities for companies to invest in emission reduction, particularly if a tax policy or a cap-and-trade policy is in place.

The second set of results leads to the following conclusion:

- In comparison to the case where there is no emission regulation in place, the cap policy and the tax policy reduce annual carbon emissions at the expense of increased annual total costs. (If the cap is not binding, annual costs and emissions do not change under the cap policy.) On the other hand, it is possible to reduce carbon emissions with decreased annual total costs under a cap-and-trade policy.

In the next two lemmas, we present some results following a direct comparison of the different regulation policies.

LEMMA 10 *For any tax policy with parameter $p > 0$, a better cap policy can be designed by an appropriate choice of parameter $C > 0$ so that $TC_1(Q_1^*, G_1^*) < TC_2(Q_2^*, G_2^*)$ and $E(Q_1^*, G_1^*) \leq E(Q_2^*, G_2^*)$. On the other hand, for a cap policy with parameter $C > 0$, a better tax policy with parameter $p > 0$ cannot be found to result in $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$ and $E(Q_2^*, G_2^*) \leq E(Q_1^*, G_1^*)$.*

Lemma 10 indicates that for any tax policy, it is possible to design a lower-cost cap policy for the retailer without increasing his/her emissions levels. Therefore, as far as the resulting costs and emissions of the retailer are concerned, policy-makers may prefer a cap policy over a tax policy. In the next lemma, we present the result of a similar comparison between the cap policy and the cap-and-trade policy.

LEMMA 11 *Consider a cap policy with parameter $C > 0$, and a cap-and-trade policy with parameters $C > 0$ and $c_p > 0$. We have $TC_3(Q_3^*, G_3^*) \leq TC_1(Q_1^*, G_1^*)$ for any value of c_p . Furthermore, given a value of the common parameter C , there exists a positive value of c_p such that $E(Q_3^*, G_3^*) \leq E(Q_1^*, G_1^*)$.*

Lemma 11 implies that corresponding to every cap policy, there exists a cap-and-trade policy with lower carbon emissions and lower costs per unit time for the retailer if the value of the carbon trading price is right. Lemmas 10 and 11 together imply that given a tax policy it is possible to have

$$TC_3(Q_3^*, G_3^*) \leq TC_1(Q_1^*, G_1^*) \leq TC_2(Q_2^*, G_2^*)$$

with appropriate values of C and c_p .

5. Numerical analysis

In this section, we present the results of a numerical study to further investigate how the retailer's annual costs and emissions change with respect to the policy parameters, and how the investment option and its parameters affect the annual costs and emissions under each policy. In addition to $TC_i(Q_i^*, G_i^*)$ and $E_i(Q_i^*, G_i^*)$, we define a new measure to assess the increase in costs relative to the decrease in emissions. We refer to this measure as *cost of unit emission reduction* and we define it as follows for policy i

$$\frac{TC_i(Q_i^*, G_i^*) - TC(Q^0, 0)}{E(Q^0, 0) - E(Q_i^*, G_i^*)}.$$

It is important to note that some of our analytical results in Section 3 provide general explanations to the issues that are brought up in this section more explicitly. Our numerical analysis complements these findings, particularly where only limited analytical results were possible. Because the solution under the cap policy as given in Theorem 1 is more complex than those under the tax and the cap-and-trade policies, it has been possible to obtain more analytical results involving the latter two policies. Therefore, it is no coincidence that more of the numerical results in this section concern the cap policy.

Our analysis in Section 3 reveals that how $\frac{A}{h}$ compares to $\frac{\hat{A}}{\hat{h}}$ is an important characteristic of the setting that affects the solutions under all three policies. Therefore, our analysis considers two sets of instances: one with $A = 100$, $h = 3$, $\hat{A} = 4$ and $\hat{h} = 3$, and the other with $A = 10$, $h = 4$, $\hat{A} = 100$ and $\hat{h} = 8$. Here, we have $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ in the first set of instances and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ in the second set of instances. In all instances, we take $D = 500$, $c = 6$, and $\hat{c} = 2$. In what follows, we first present our results for the cap policy, then we proceed with our findings on the tax and cap-and-trade policies.

5.1 Numerical study for cap policy

In this section, we present the results of our numerical study on cap policy with two main objectives: first, to characterise how the annual costs, savings achieved by investment and the cost of unit emission reduction change under different values of the policy parameter C , and secondly, to gain insights on how the retailer makes a choice between two investment options with different parameters.

Figure 1(a) shows an illustration of how $TC_1(Q_1^*, G_1^*)$ changes with respect to varying values of C for the case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. Figure 1(b) is a similar plot for the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. The resulting annual cost and emission levels for some specific instances under three scenarios (i.e. cap policy, cap policy without investment and no-policy) are also presented in Table 2. It can be observed from Figure 1(a) and (b) that starting from the smallest possible values of C (based on Expression (6)), $TC_1(Q_1^*, G_1^*)$ first exhibits a strictly decreasing pattern with respect to increasing values of C , and then, the costs level in both figures. The value of C after which annual costs become constant coincides with $E(Q^0, 0)$. If $C \geq E(Q^0, 0)$, then the cap is no longer restrictive, and the solution to the retailer's problem under no emission policy optimises his/her costs under the cap policy as well. As a result, in both figures, $TC_1(Q_1^*, G_1^*)$ ranges from $TC_1(Q^e, \frac{\alpha}{2\beta})$ to $TC_1(Q^0, 0)$. It can also be observed from both figures that a one-unit decrease in the cap is more costly to the retailer at its already small values.

Table 2 reports some instances to illustrate the possible different solution types to the retailer's problem under the cap policy, as given in Theorem 1. In the first set of instances, characterised by $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, $Q_1^* = Q^0$ and $G_1^* = 0$ for $C \geq 1284.816$. Similarly, in the second set of instances, $Q_1^* = Q^0$ and $G_1^* = 0$ for $C \geq 2200$. For those values of C that are large enough (i.e. $C \geq 1284.816$ and $C \geq 2200$ in the first and second sets, respectively), having a cap policy does not change the solution in comparison to a no-policy case because the cap amount is not restrictive. Therefore, we have $TC_1(Q_1^*, G_1^*) = TC_1(Q_1^*(0), 0) = TC(Q^0, 0)$ in such instances. In the third instances of each set ($C = 1270$ and $C = 2110$ in the first and the second sets, respectively), we have $TC(Q^0, 0) < TC_1(Q_1^*, G_1^*) = TC_1(Q_1^*(0), 0)$ and $E(Q^0, 0) > E(Q_1^*, G_1^*) = E(Q_1^*(0), 0)$. Here, the cap policy helps to decrease emissions at the expense of increased costs, and the retailer does not invest in new technology to further reduce emissions even if such an option exists. In the second instances of each set ($C = 1170$ and $C = 1910$ in the first and the second sets, respectively), we have

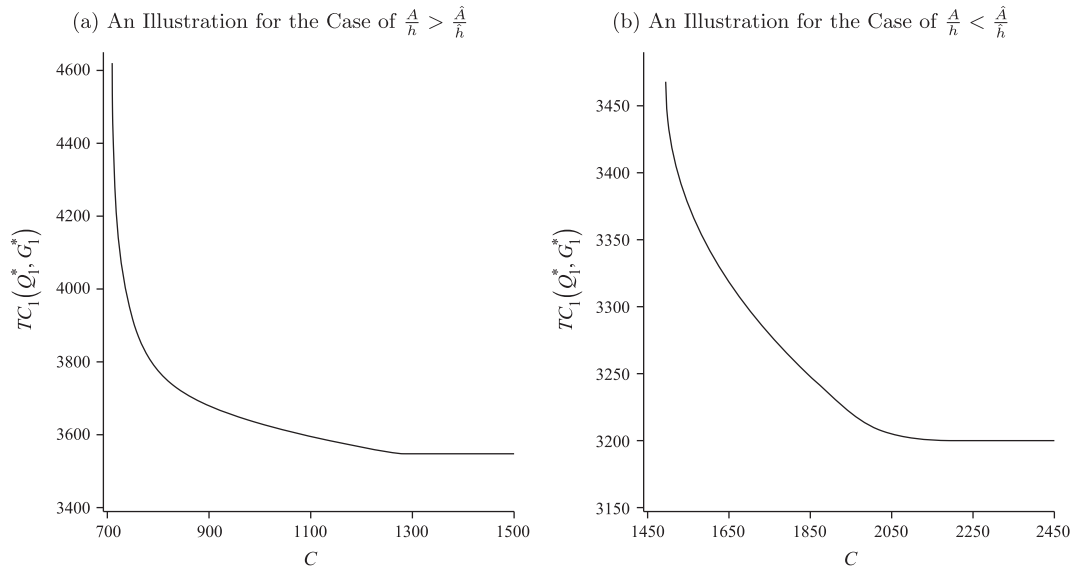


Figure 1. Behaviour of $TC_1(Q_1^*, G_1^*)$ for varying values of C under a cap policy.

Table 2. Numerical illustrations under the cap policy for varying values of the cap given $\alpha = 4$ and $\beta = 0.01$.

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
Instances with $\frac{A}{h} > \frac{\hat{A}}{h}$ ($Q^0 = 182.574$, $Q^e = 36.515$, $Q^\alpha = 164.114$, $E(Q^0, 0) = 1284.816$, $TC(Q^0, 0) = 3547.723$)									
1070	—	—	—	158.904	51.994	1070	3605.005	—	—
1170	100	13.333	100	162.127	22.666	1170	3574.257	1170	3650
1270	172.26	7.74	172.26	172.26	0	1270	3548.649	1270	3548.649
1370	241.137	5.529	182.574	182.574	0	1284.816	3547.723	1284.816	3547.723
Instances with $\frac{A}{h} < \frac{\hat{A}}{h}$ ($Q^0 = 50$, $Q^e = 111.803$, $Q^\alpha = 76.376$, $E(Q^0, 0) = 2200$, $TC(Q^0, 0) = 3200$)									
1710	—	—	—	82.556	68.043	1710	3293.72	—	—
1910	134.704	92.796	92.796	77.283	11.879	1910	3231.142	1910	3239.474
2110	220.918	56.582	56.582	56.582	0	2110	3201.531	2110	3201.531
2310	283.391	44.109	50	50	0	2200	3200	2200	3200

$TC(Q^0, 0) < TC_1(Q_1^*, G_1^*) < TC_1(Q_1^*(0), 0)$ and $E(Q^0, 0) > E(Q_1^*, G_1^*) = E(Q_1^*(0), 0)$. Again, the cap policy reduces annual emissions and increases annual costs, but different than the third instances, the investment option helps to achieve the same emissions at lower costs in comparison to no investment opportunity. Finally, the first instances of each set are illustrative of situations in which it is not possible to be within the allowed emission limits without making an investment.

In Lemma 1, we have shown that $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*) \geq 0$. The exact value of $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*)$ is a measure of the savings due to the investment opportunity under the cap policy. Figure 2 illustrates how this difference changes with respect to C for the cases of $\frac{A}{h} > \frac{\hat{A}}{h}$ and $\frac{A}{h} < \frac{\hat{A}}{h}$. In both cases, values of C for which $Q_1^*(0)$ exists are considered. As a result, we have $C \geq 1109.545$ in Figure 2(a) and $C \geq 1894.427$ in Figure 2(b). Observe also that the savings due to the investment opportunity are more significant at tight values of the cap. Furthermore, the retailer no longer uses the investment opportunity (i.e. $G_1^* = 0$) if C is greater than or equal to $E(Q^0, 0)$.

Figure 3(a) and (b) illustrates how the cost of unit emission reduction changes for varying values of the cap in cases of $\frac{A}{h} > \frac{\hat{A}}{h}$ and $\frac{A}{h} < \frac{\hat{A}}{h}$, respectively. We know from Lemma 2 that $E(Q_1^*, G_1^*) \leq E(Q^0, 0)$. Both figures are plotted for those values of C at which $E(Q_1^*, G_1^*) < E(Q^0, 0)$. Mainly, Figure 3(a) considers values of C up to 1284.816 and Figure 3(b) considers values of C up to 2200. Observe that in both cases, reducing the annual emission level by one unit is more costly at small values of C . Furthermore, in case of $\frac{A}{h} > \frac{\hat{A}}{h}$, the cost of a one-unit emission increases more rapidly as C gets smaller in comparison to the case of $\frac{A}{h} < \frac{\hat{A}}{h}$.

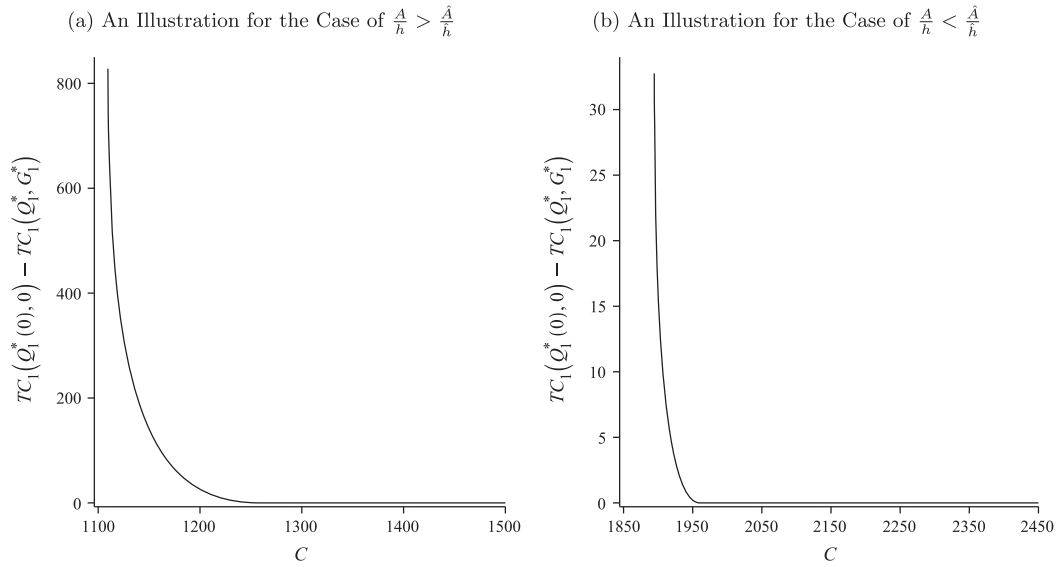


Figure 2. Savings due to an investment opportunity for varying values of the cap under a cap policy.

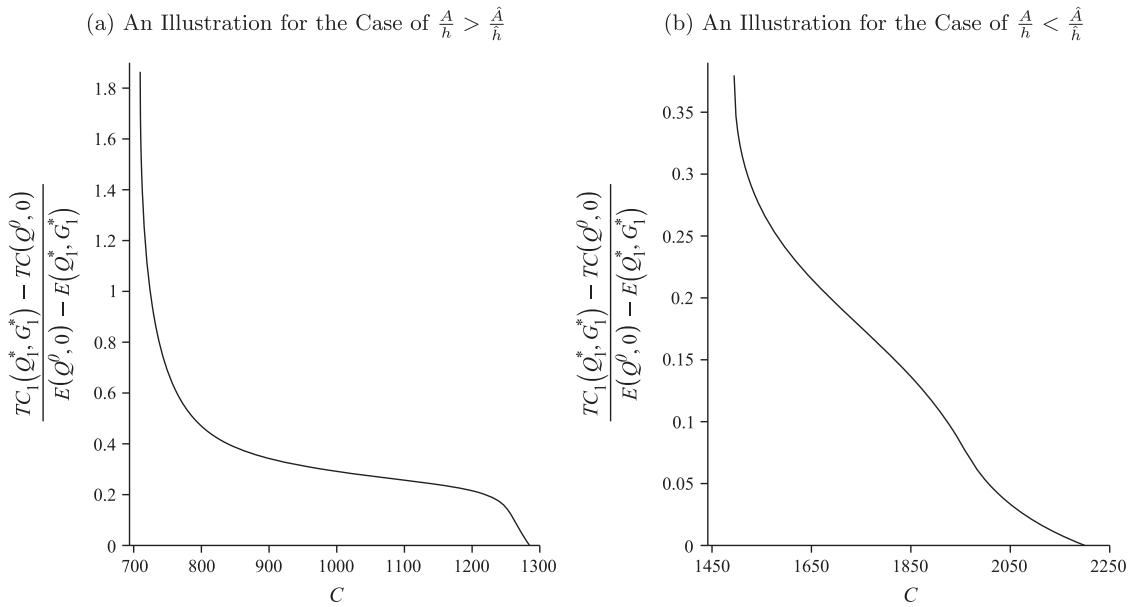


Figure 3. Cost of unit emission reduction for varying values of the cap under a cap policy.

In Lemma 3, we have shown that among two investment options with different parameters, the retailer should choose the one with higher α and smaller β . In Figure 4, we show over numerical examples that if the investment option with higher α does not have smaller β , whether it is a better investment option or not depends on how high the α value is. Specifically, in Figure 4(a), for the case of $\frac{A}{h} > \frac{\hat{A}}{h}$, setting $C = 840$, $\alpha_1 = 9.4$, $\beta_1 = 0.02$ and $\beta_2 = 0.02$, we change the value of α_2 and track the difference between the minimum annual costs resulting from the two investment options. $TC_1(Q_1^*, G_1^* | \alpha_1 = 9.4, \beta_1 = 0.02)$ refers to the minimum costs, given that the first investment option has parameters $\alpha_1 = 9.4$ and $\beta_1 = 0.02$. Similarly, $TC_1(Q_1^*, G_1^* | \alpha_2, \beta_2 = 0.025)$ denotes the minimum costs if the second investment option has a value of α_2 as given on the x-axis, and $\beta_2 = 0.025$. Figure 4(a) shows that for all values of $\alpha_2 < 9.656$, the first investment option has lower costs. As α_2 increases beyond this value, the second investment option becomes more preferable. Figure 4(b) illustrates a similar result for the case of $\frac{A}{h} < \frac{\hat{A}}{h}$, setting $C = 1700$, $\alpha_1 = 12.3$, and $\beta_1 = 0.02$, $\beta_2 = 0.025$. The second investment option becomes better as α_2 is increased beyond 12.445. Notice that for values of α_2 between 12.3 and 12.445, the second investment option still has higher α and higher β , yet the first investment option leads to lower annual costs.

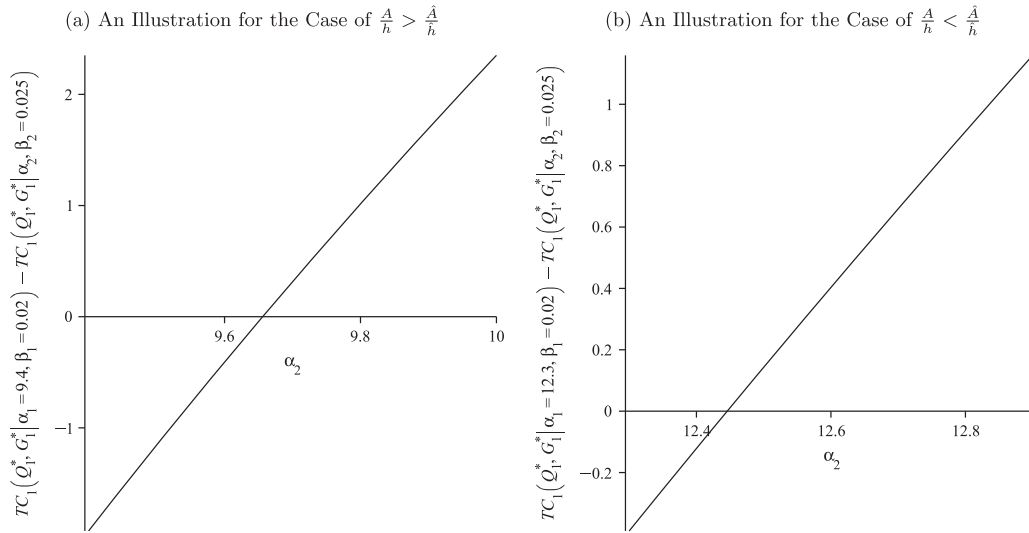


Figure 4. Comparison of costs under two different investment options in case of a cap policy.

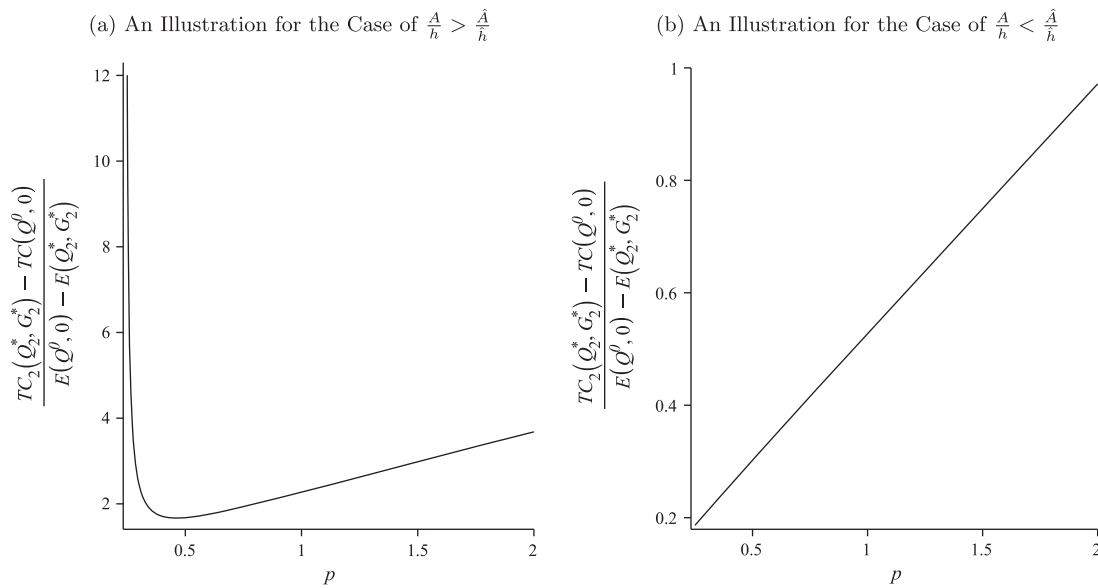


Figure 5. Cost of unit emission reduction for varying values of tax under a tax policy.

5.2 Numerical study for tax policy and cap-and-trade policy

Corollary 2 and Lemma 4 provide analytical results for $TC_2(Q_2^*, G_2^*)$ and $TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*)$, which imply that both measures are increasing in p . In our numerical analysis for the tax policy, then, we proceed with investigating the effect of policy parameter p on the cost of unit emission reduction (i.e. $\frac{TC_2(Q_2^*, G_2^*) - TC(Q^0, 0)}{E(Q^0, 0) - E(Q_2^*, G_2^*)}$). In Figure 5(a), which pertains to the case of $\frac{A}{h} > \frac{\hat{A}}{h}$, the cost of unit emission reduction is strictly convex in p , with a minimum at $p = 0.463$. In our numerical experimentation with various instances having $\frac{A}{h} < \frac{\hat{A}}{h}$, we observe that $\frac{TC_2(Q_2^*, G_2^*) - TC(Q^0, 0)}{E(Q^0, 0) - E(Q_2^*, G_2^*)}$ assumes a shape similar to the one in Figure 5(a). In Figure 5(b), for the case of $\frac{A}{h} < \frac{\hat{A}}{h}$, we change the value of \hat{A} to 1000 to illustrate an extreme situation where the cost of unit emission reduction increases almost linearly with increasing p over all its possible values.

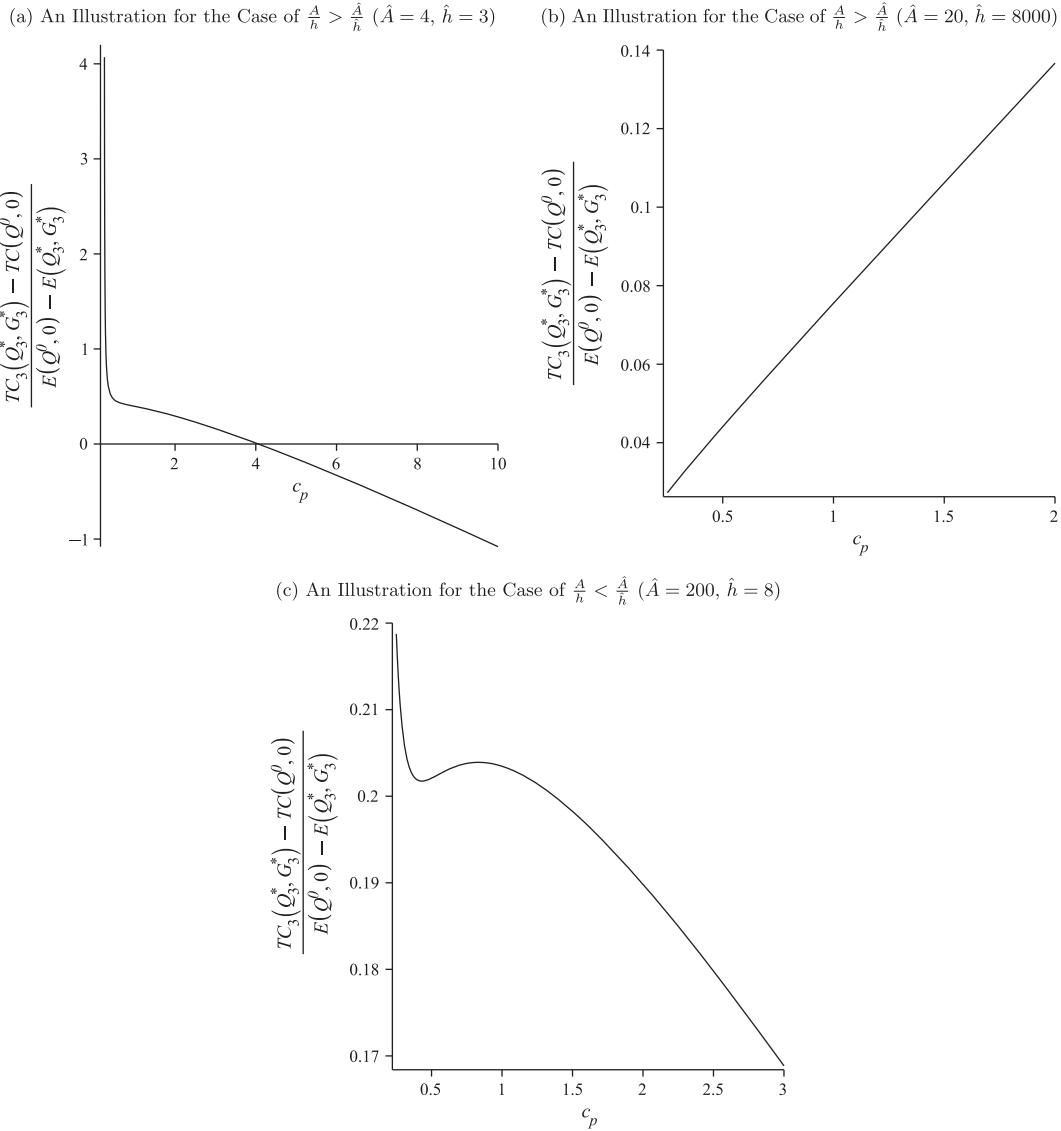


Figure 6. Cost of unit emission reduction for varying values of the trading price under a cap-and-trade policy.

As in the case of the tax policy, our numerical analysis for the cap-and-trade policy focuses on investigating how the cost of unit emission reduction changes with respect to policy parameters. Corollary 4 and Lemma 8 provide analytical results for $TC_3(Q_3^*, G_3^*)$ and $TC_3(Q_3^*(0), 0) - TC_3(Q_3^*, G_3^*)$. Figure 6 presents three different illustrations of how the cost of unit emission reduction behaves with changing values of c_p . In the examples underlying Figure 6(a) and (c), there exist values of c_p ($c_p \geq 0.9754$ in Figure 6(a) and $c_p \geq 1.148$ in Figure 6(c)) at which the retailer sells his/her cap. In both of these examples, as c_p increases beyond these values, $TC_3(Q_3^*, G_3^*)$ gets smaller and smaller due to the revenue earned from selling permits. $TC_3(Q_3^*, G_3^*)$ falls below $TC(Q^0, 0)$ when $c_p \geq 4.061$ and when $c_p \geq 9.75$ in the examples of Figure 6(a) and (c), respectively. Figure 6(b) illustrates an example to Corollary 3. Because the retailer does not sell any carbon permits, regardless of the value of c_p , $TC_3(Q_3^*, G_3^*)$ is always greater than $TC_3(Q_3^*(0), 0)$. Furthermore, as c_p increases, the cost of unit emission reduction increases.

5.3 Numerical comparison of the three policies

In Section 4, we proved that for any tax policy, there exists a cap policy with lower annual costs and no greater annual emissions. Similarly, for any cap policy, there exists a cap-and-trade policy with no greater annual costs and no greater annual emissions. In this subsection, we investigate how the differences between the annual costs and the annual emissions of any two policies change with respect to the problem parameters.

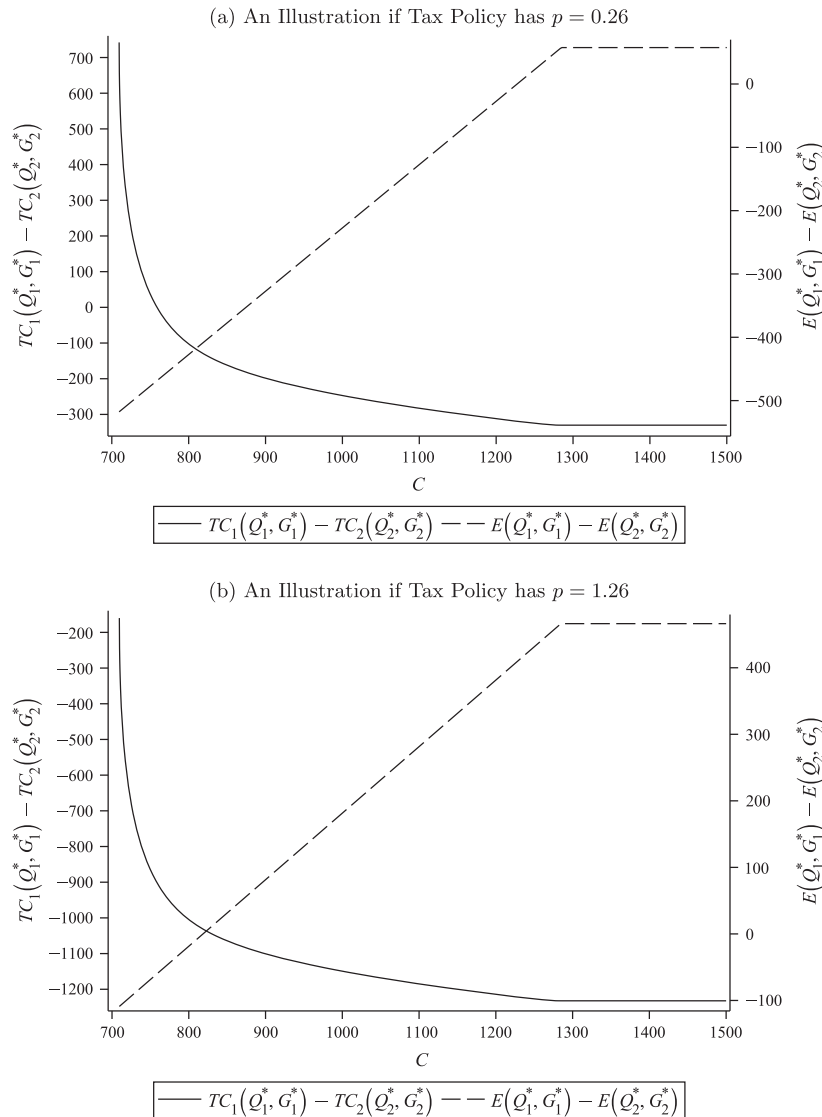


Figure 7. Comparison of tax policy to cap policy for annual costs and annual emissions.

In Figure 7, we present two illustrations for the comparison of the cap and tax policies in a setting with parameters $A = 100, h = 3, \hat{A} = 4, \hat{h} = 3, \alpha = 4$ and $\beta = 0.01$. Figure 7(a) shows a plot of how $TC_1(Q_1^*, G_1^*) - TC_2(Q_2^*, G_2^*)$ and $E(Q_1^*, G_1^*) - E(Q_2^*, G_2^*)$ simultaneously change for varying values of C , given that the tax policy has $p = 0.26$. For values of C lower than 758.832, the tax policy is better in terms of annual costs. As C increases beyond this value, the cap policy becomes better in terms of annual costs and annual emissions up until $C = 1227.296$. For C values larger than 1227.296, the cap policy is more advantageous because of its resulting costs, however the tax policy is better because of its resulting emissions. Figure 7(b) presents a similar plot, given that tax policy has $p = 1.26$. At all values of C , the cap policy is more advantageous for the retailer because of its resulting costs. However, the tax policy leads to lower annual emissions for the retailer in comparison to any cap policy with parameter $C \geq 818.520$. Observe from both Figure 7(a) and (b) that there is no value of C at which the tax policy is better for both its costs and its emissions, as also implied by Lemma 10.

For the same setting underlying Figure 7, we next compare the cap policy to the cap-and-trade policy. We consider two different values of c_p for the latter: 0.26 and 1.26. Figure 8(a) shows how $TC_1(Q_1^*, G_1^*) - TC_2(Q_2^*, G_2^*)$ and $E(Q_1^*, G_1^*) - E(Q_2^*, G_2^*)$ simultaneously change with varying values of C when $c_p = 0.26$. At all values of C , the cap-and-trade policy leads to lower annual costs, however, the cap policy results in lower annual emissions than the cap-and-trade does if $C < 1227.296$. Otherwise, the cap-and-trade policy is also better in terms of annual emissions. Similarly, Figure 8(b) shows that cap-and-trade policy is more advantageous for the retailer because of its resulting costs at all values of C ; however, the

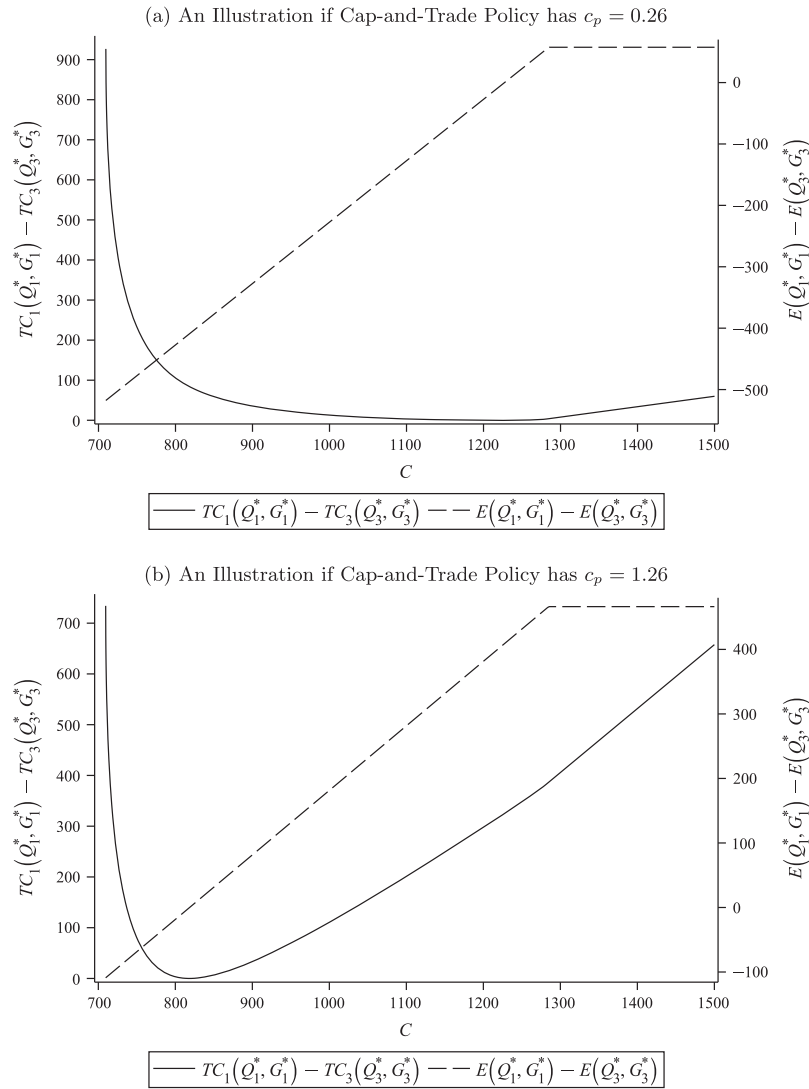


Figure 8. Comparison of cap policy to cap-and-trade policy for annual costs and annual emissions.

dominance of one policy over another in terms of annual emissions changes depending on the value of C . Specifically, if $C \geq 818.520$, then the cap-and-trade policy dominates in terms of both measures, otherwise, the cap policy leads to lower annual emissions for the retailer.

6. Conclusion

In this paper, we study a retailer’s joint decisions on inventory replenishment and emission reduction investment operating under the conditions of the classic EOQ model. We consider three emission regulation policies; cap, tax and cap-and-trade. Our results provide guidelines and insights about five issues: (i) how much the retailer should order at each replenishment and how much he/she should invest in emission reduction to minimise long-run average costs, (ii) what the impact of having an investment option is on the retailer’s annual costs and emissions, (iii) how the retailer’s annual costs and emissions under an emission regulation policy compare to those when no regulation is in place, (iv) how the retailer should choose among different investment options available and (v) how the different regulation policies compare in terms of the retailer’s annual emissions and costs.

Analytical expressions for the optimal replenishment quantity and investment amount for the cap policy, tax policy and cap-and-trade policy are presented in Theorems 1–3, respectively. Our findings imply that an investment option may help the retailer to reduce his/her costs significantly under all policies; however, the retailer’s annual emissions level does not

decrease due to investing in case of the cap policy. Under the tax policy and the cap-and-trade policy, the retailer always takes advantage of the investment opportunity to further reduce his/her emissions, which implies that there is better motivation for governments to make investment opportunities available under the tax or cap-and-trade policy. We have not modelled the possibility that the company invents his/her own environmental technologies, but these results also suggest that it is more likely that a company involves in technological innovation in case of tax or cap-and-trade policies rather than in a cap policy.

When carefully designed, all three regulation policies are effective in reducing carbon emissions. The cap and tax policies always lead to higher annual costs for the retailer compared to when no regulation policy is in place. On the other hand, under a cap-and-trade policy, the retailer may reduce his/her costs by selling permits equivalent to his/her excess carbon capacity. For the retailer not to profit solely from selling permits, there must exist an upper bound on the maximum annual carbon emission (see Corollary 5).

The investment function considered in this study has a nonlinear form characterised by two parameters. Lemmas 3, 6 and 9 provide guidelines in terms of those parameters on how the retailer should choose among different investment options. Our results imply that in case of the cap policy, the right choice of investment opportunity may help the retailer further reduce his/her annual costs, but it does not have an impact on annual emissions. We show that a better investment opportunity for reducing costs may lead to more annual emissions in some cases under a tax policy or a cap-and-trade policy. We also characterise the cases in which it is possible to reduce both the annual costs and the annual emissions by the right choice of investment opportunity.

We also show that for a given cap or cap-and-trade policy, it is not possible to design a tax policy that leads to both lower costs and lower emissions. On the other hand, for a given tax policy, a better cap policy can be designed by the appropriate choice of cap value. Further, for a given cap policy, there may exist a cap-and-trade policy that is better for both the resulting costs and the resulting emissions.

In our numerical analysis, we have defined a measure that we refer to as ‘cost of unit emission reduction’. This measure is the ratio of the cost increase to the savings in emissions, and its value for a certain policy can be considered as the *social cost* of that policy. We have observed that the social cost becomes very high as the policy parameters are tightened in case of cap and tax policies (i.e. annual carbon emission cap is decreased in cap policy, or tax paid for one unit emission is increased in tax policy). In fact, the increase in social cost is more emphasised when the company’s ratio of fixed cost of replenishment to his/her inventory holding cost rate is very high (i.e. $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ in terms of problem parameters). This suggests, in an inventory setting under a cap or a tax policy, reducing the company’s ordering costs along with green technologies may decrease the social cost. However, we believe further research is needed to explore what kind of production/inventory related parameters are of significance in reducing cost of compliance to emission regulations. Our numerical analysis (see Figure 6) shows that a cap-and-trade policy, considering the measure of cost of unit emission reduction, may sometimes be rewarding (other times costly) depending on whether the company is able to generate excess carbon allowance to sell or not.

The use of a quadratic emission reduction function has made it possible to obtain analytical results that lead to the implications as discussed above. An important characteristic of this function, which we have utilised extensively in our analysis, is that it is a concave, increasing function until a certain value of investment (i.e. $\alpha/(2\beta)$). The increasing behaviour of the function upto a certain point shows that investment in green technology is efficient in reducing emissions, but there is a maximum potential of abatement. The concavity implies that it becomes more costly to reduce emissions as emissions are decreased (the low hanging fruit has been picked). The analytical expressions we have derived, naturally depend on the parameters of this function, however, we believe our general conclusions still hold in case of other investment functions which exhibit these characteristics.

Our model assumes a single item. An immediate extension would be to study the joint decisions for replenishment and allocation of limited investment budget (for emission reduction) among multiple items to maximise the profits. Our study also considers a retailer operating under the conditions of the EOQ model, which is one of the fundamental models of inventory theory. As we have mentioned in the introduction, there is a new paradigm in inventory theory (i.e. inventory planning to help the environment) that revisits classical models to take into account issues related to energy, environment, waste disposal, etc. (e.g. Hasanov et al. 2013; Jaber 2009; Sarkis, Zhu, and Lai 2011). The questions raised in this paper can also be investigated for settings with different inventory replenishment policies.

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Appendix 1.

A.1 Proof of Theorem 1

The proof will follow by making use of the Karush–Kuhn–Tucker (KKT) conditions. The objective function is differentiable, and it is convex because its Hessian matrix $\begin{pmatrix} \frac{2AD}{Q^3} & 0 \\ 0 & 0 \end{pmatrix}$ is positive semi-definite. Emission cap constraint is also differentiable, and it is strictly convex in Q and G because its Hessian matrix $\begin{pmatrix} \frac{2\hat{A}D}{Q^3} & 0 \\ 0 & 2\beta \end{pmatrix}$ is positive definite. In addition, Assumption (A4) implies that there exists a feasible point in the set $\left\{ \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 < C, Q \geq 0, G \geq 0 \right\}$. As a result, we conclude that the KKT conditions listed

below guarantee global optimality along with feasibility conditions.

$$\frac{-AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(\frac{-\hat{A}D}{Q^2} + \frac{\hat{h}}{2} \right) - \mu_1 = 0, \quad (14)$$

$$1 + \lambda_1(-\alpha + 2\beta G) - \mu_2 = 0, \quad (15)$$

$$\lambda_1 \left(C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D + \alpha G - \beta G^2 \right) = 0, \quad (16)$$

$$\mu_1 Q = 0, \quad (17)$$

$$\mu_2 G = 0, \quad (18)$$

$$\lambda_1 \geq 0, \quad \mu_1 \geq 0, \quad \mu_2 \geq 0. \quad (19)$$

The multipliers λ_1 , μ_1 , and μ_2 may be equal to zero or be greater than zero. Considering these alternatives, there are eight possible cases, however, only the following three may lead to feasible solutions.

Case 1 $\lambda_1 = 0, \mu_1 = 0, \mu_2 > 0$

Expressions (16) and (17) are satisfied because $\lambda_1 = 0$ and $\mu_1 = 0$. Expression (15) implies $\mu_2 = 1$. Because $\mu_2 > 0$, Expression (18) leads to $G = 0$. Finally, evaluating Expression (14) at $\lambda_1 = 0$ and $\mu_1 = 0$, we obtain $Q = Q^0 = \sqrt{\frac{2AD}{h}}$.

Now, let us check the feasibility of $Q = \sqrt{\frac{2AD}{h}}$ and $G = 0$. When $G = 0$, to find a feasible order quantity, we should have $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, because the contrary implies that even the minimum carbon emission possible by ordering decisions would exceed the emission cap. In addition, any feasible order quantity Q should satisfy $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \leq C$. This inequality further yields $Q_2 \leq Q \leq Q_1$, where Q_1 and Q_2 are defined in (7) and (8). Observe that since $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, both Q_1 and Q_2 exist. Therefore, if $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ and $Q_2 \leq Q^0 \leq Q_1$, then $Q_1^* = Q^0$ and $G_1^* = 0$.

Case 2 $\lambda_1 > 0, \mu_1 = 0, \mu_2 > 0$

Using the fact that $\mu_1 = 0$, Expression (14) can be rewritten as

$$-\frac{AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(-\frac{\hat{A}D}{Q^2} + \frac{\hat{h}}{2} \right) = 0. \quad (20)$$

Since $\mu_2 > 0$, Expression (18) implies $G = 0$. Therefore, Expression (15) reduces to

$$1 - \alpha\lambda_1 - \mu_2 = 0. \quad (21)$$

Because $\lambda_1 > 0$ and $G = 0$, Expression (16) implies

$$C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D = 0.$$

Note that, Q_1 and Q_2 are the two values of Q that satisfy the above equality. Since $G = 0$, we should have $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ for the same reason as discussed in Case 1, which in turn, implies that Q_1 and Q_2 exist. In the rest of our analysis for Case 2, we will consider the following two possibilities:

Case 2.1 $C = \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$

It can be shown that if $C = \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, then $Q_1 = Q_2 = \sqrt{\frac{2\hat{A}D}{h}}$. In this case, Expression (20) holds for any positive value of λ_1 as long as $\frac{A}{h} = \frac{\hat{A}}{h}$. However, due to the relationship between λ_1 and μ_2 as stated in Expression (21) and the fact that $\mu_2 > 0$, λ_1 should be chosen such that $\lambda_1 < \frac{1}{\alpha}$. Therefore, if $\frac{A}{h} = \frac{\hat{A}}{h}$, then $Q_1^* = Q^0$ and $G_1^* = 0$.

Case 2.2 $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$

If $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, then $Q_1 \neq Q_2$. For $Q = Q_1$ or $Q = Q_2$ to be optimal, there must exist positive values of λ_1 and μ_2 that satisfy Expressions (20) and (21). Using Expression (20), we obtain

$$\lambda_1 = \frac{\frac{AD}{Q^2} - \frac{h}{2}}{-\frac{\hat{A}D}{Q^2} + \frac{\hat{h}}{2}} = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}.$$

Note that, since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, it turns out that the denominator of the above expression is different than zero for $Q = Q_1$ and $Q = Q_2$, therefore, λ_1 is finite. Utilising this expression in (21) further leads to

$$\mu_2 = 1 - \alpha \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}.$$

Since $\lambda_1 > 0$ and $\mu_2 > 0$, any optimal Q should then satisfy

$$0 < \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2} < \frac{1}{\alpha}. \quad (22)$$

Now, let us check the conditions for Q_1 to satisfy the above expression, and hence, to be optimal. Since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, we have

$$2(C - \hat{c}D)^2 - 4\hat{A}D\hat{h} > 0.$$

Combining $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ with the fact that $\sqrt{2\hat{A}D\hat{h}} > 0$, we conclude

$$2(C - \hat{c}D)^2 + 2(C - \hat{c}D)\sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} - 4\hat{A}D\hat{h} > 0,$$

which can be rewritten as

$$\left[C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} \right]^2 - 2\hat{A}D\hat{h} > 0.$$

The above inequality implies

$$-2\hat{A}D + \hat{h} \frac{\left[C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} \right]^2}{\hat{h}^2} > 0.$$

Observe from Expression (7) that, the fractional term in the above expression is equal to Q_1^2 , therefore, we have

$$-2\hat{A}D + \hat{h}Q_1^2 > 0.$$

Based on the above result, for Expression (22) to hold for $Q = Q_1$, we should have $2AD - hQ_1^2 > 0$ and $\frac{2AD - hQ_1^2}{-2\hat{A}D + \hat{h}Q_1^2} < \frac{1}{\alpha}$. Evaluating these two expressions, we conclude that if $Q_1 < Q^0 = \sqrt{\frac{2AD}{h}}$ and $Q_1 > Q^\alpha = \sqrt{\frac{2(\hat{A} + A\alpha)D}{\hat{h} + h\alpha}}$, then $Q_1^* = Q_1$ and $G_1^* = 0$.

To check the conditions for optimality of Q_2 , we use a similar methodology. Since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, we have

$$\left((C - \hat{c}D)^2 - 2\hat{A}D\hat{h} \right)^2 < (C - \hat{c}D)^2 \left((C - \hat{c}D)^2 - 2\hat{A}D\hat{h} \right),$$

which, in turn, implies that

$$(C - \hat{c}D)^2 - 2\hat{A}D\hat{h} - (C - \hat{c}D)\sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} < 0.$$

Multiplying both sides of the above expression with $\frac{2}{\hat{h}}$ leads to

$$-2\hat{A}D + \hat{h} \frac{\left[(C - \hat{c}D) - \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} \right]^2}{\hat{h}^2} < 0.$$

Observe from Expression (8) that, the fractional term in the above expression is equal to Q_2^2 , therefore, we have

$$-2\hat{A}D + \hat{h}Q_2^2 < 0.$$

Based on the above result, for Expression (22) to hold for $Q = Q_2$, we should have $2AD - hQ_2^2 < 0$ and $\frac{2AD - hQ_2^2}{-2\hat{A}D + \hat{h}Q_2^2} < \frac{1}{\alpha}$. Evaluating these two expressions, we conclude that if $Q_2 > Q^0 = \sqrt{\frac{2AD}{h}}$ and $Q_2 < Q^\alpha = \sqrt{\frac{2(\hat{A} + A\alpha)D}{\hat{h} + h\alpha}}$, then $Q_2^* = Q_2$ and $G_2^* = 0$.

Case 3 $\lambda_1 > 0, \mu_1 = 0, \mu_2 = 0$

Expression (17) and Expression (18) are satisfied because $\mu_1 = 0$ and $\mu_2 = 0$. Using the fact that $\mu_1 = 0$, Expression (14) can be rewritten as

$$\frac{-AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(\frac{-\hat{A}D}{Q^2} + \frac{\hat{h}}{2} \right) = 0. \quad (23)$$

Since $\mu_2 = 0$, Expression (17) reduces to

$$1 + \lambda_1(-\alpha + 2\beta G) = 0. \quad (24)$$

As $\lambda_1 > 0$, Expression(16) implies

$$\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - C - \alpha G + \beta G^2 = 0. \tag{25}$$

Now, we should find non-negative values of Q and G , and a positive value of λ_1 that solve the system of equations as given by (23), (24) and (25). It follows from Expression (24) that $G < \frac{\alpha}{2\beta}$. For any value of G , Expression (25) is satisfied at the following two values of Q , which we refer to as $Q_3(G)$ and $Q_4(G)$:

$$Q_3(G) = \frac{(C - \hat{c}D + \alpha G - \beta G^2) + \sqrt{(C - \hat{c}D + \alpha G - \beta G^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \tag{26}$$

$$Q_4(G) = \frac{(C - \hat{c}D + \alpha G - \beta G^2) - \sqrt{(C - \hat{c}D + \alpha G - \beta G^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \tag{27}$$

For the existence of such $Q_3(G)$ and $Q_4(G)$, we should have $C - \hat{c}D + \alpha G - \beta G^2 \geq \sqrt{2\hat{A}\hat{h}D}$. In the rest of our analysis for Case 3, we will consider the following two possibilities:

Case 3.1 $C - \hat{c}D + \alpha G - \beta G^2 = \sqrt{2\hat{A}\hat{h}D}$

In this case, $Q_3(G) = Q_4(G) = Q^e = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$. When $Q = Q^e$, Expression (23) holds for any $\lambda_1 > 0$ as long as $\frac{\hat{A}}{h} = \frac{A}{h}$. Now, for any value of G that satisfies $C - \hat{c}D + \alpha G - \beta G^2 = \sqrt{2\hat{A}\hat{h}D}$ to be optimal, we should have $0 \leq G < \frac{\alpha}{2\beta}$. Although there are two real roots of this equation, these conditions only hold at $G = G_5 = \frac{\alpha - \sqrt{\alpha^2 - 4\beta(-C + \hat{c}D + \sqrt{2\hat{A}\hat{h}D})}}{2\beta}$. Therefore, if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $\frac{\hat{A}}{h} = \frac{A}{h}$, then $Q_1^* = Q^e$ and $G_1^* = G_5$.

Case 3.2 $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}\hat{h}D}$

If $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}\hat{h}D}$, then $Q_3(G) \neq Q_4(G)$. For any $(Q_3(G), G)$ or $(Q_4(G), G)$ pair to be optimal, there must exist corresponding positive values of λ_1 that satisfy Expression (20). That is, we should have $\lambda_1 = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2} > 0$. Now, let us check the conditions for $Q_3(G)$ to satisfy this inequality. It can be shown that $-2\hat{A}D + \hat{h}Q_3^2(G) > 0$, or equivalently $Q_3(G) > Q^e$, for any given value of G that satisfies $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}\hat{h}D}$. Combining the condition of having $\lambda_1 > 0$ with the fact that $-2\hat{A}D + \hat{h}Q_3^2(G) > 0$, we conclude $2AD - hQ_3^2(G) > 0$. This implies $Q_3(G) < Q^0$.

Next, utilising $\lambda_1 = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}$ in Expression (24), we obtain

$$G = \frac{2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_3^2(G)}{2\beta(2AD - hQ_3^2(G))}. \tag{28}$$

At this point, the above expression with Expression (26) lead to a unique pair of (Q, G) , which we refer to as (Q_3, G_3) . The condition that $G \geq 0$, jointly with $2AD - hQ_3^2 > 0$, implies that $2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_3^2 \geq 0$. This, in turn, leads to $Q_3 \leq Q^\alpha = \sqrt{\frac{2D(\alpha A + \hat{A})}{\alpha h + \hat{h}}}$.

We have shown that the optimality of Q_3 is due to the following conditions: $Q_3 > Q^e$, $Q_3 < Q^0$ and $Q_3 \leq Q^\alpha$. Note that $Q_3 > Q^e$ and $Q_3 < Q^0$ simultaneously hold only if $\frac{A}{h} > \frac{\hat{A}}{h}$. Having $\frac{A}{h} > \frac{\hat{A}}{h}$ further implies that $Q^\alpha < Q^0$. Therefore, we conclude that if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $Q^e < Q_3 \leq Q^\alpha$, then $Q_1^* = Q_3$ and $G_1^* = G_3$.

With a similar approach, it can be shown that (Q_4, G_4) obtained by solving Expression (27) and $G = \frac{2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_4^2(G)}{2\beta(2AD - hQ_4^2(G))}$ simultaneously, is optimal if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $Q^\alpha \leq Q_4 < Q^e$. □

A.2 Proof of Lemma 2

It follows from the expressions for $TC(Q, G)$ and $TC_1(Q, G)$, and the definition of Q^0 , that $TC(Q^0, 0) \leq TC_1(Q_1^*, 0)$. Furthermore, we have $TC_1(Q_1^*, 0) \leq TC_1(Q_1^*, G_1^*)$; thus, $TC_1(Q_1^*, G_1^*) \geq TC(Q^0, 0)$. The result about the annual emission levels follows from Corollary 1 and the fact that $E(Q^0, 0) = \frac{\sqrt{D(\hat{A}h + \hat{h}A)}}{\sqrt{2\hat{A}h}} + \hat{c}D$. □

A.3 Proof of Lemma 3

Let (\bar{Q}_2, \bar{G}_2) be the retailer's optimal solution if the second investment option (i.e. the one with parameters α_2 and β_2) is adopted. First, we will show that there exists a feasible solution under the first investment option, say (\bar{Q}_1, \bar{G}_1) , that leads to the same annual emissions

level as that of (\bar{Q}_2, \bar{G}_2) under the second investment option. Second, we will show that the annual costs at (\bar{Q}_1, \bar{G}_1) , when the first investment option is adopted, are lower than or equal to the annual costs at (\bar{Q}_2, \bar{G}_2) under the second investment option.

Let us set $\bar{Q}_1 = \bar{Q}_2$. The two conditions for (\bar{Q}_1, \bar{G}_1) along with the first investment option to lead to the same annual emissions level as that of (\bar{Q}_2, \bar{G}_2) under the second investment option are:

$$\alpha_1 \bar{G}_1 - \beta_1 (\bar{G}_1)^2 = \alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2 \quad (29)$$

and

$$\bar{G}_1 \leq \frac{\alpha_1}{2\beta_1}. \quad (30)$$

We will show that there exists a unique solution to Expression (29) that also satisfies Expression (30).

The two values of \bar{G}_1 that satisfy Expression (29) are:

$$\frac{\alpha_1 + \sqrt{(\alpha_1)^2 - 4\beta_1 (\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2)}}{2\beta_1}, \quad (31)$$

and

$$\frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1 (\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2)}}{2\beta_1}. \quad (32)$$

Note that $\frac{(\alpha_2)^2}{4\beta_2}$ is the maximum of the annual emission reduction under the second investment option. Therefore, $\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2 \leq \frac{(\alpha_2)^2}{4\beta_2}$. Since $\alpha_1 \geq \alpha_2$ and $\beta_1 \leq \beta_2$, we have $\frac{(\alpha_2)^2}{4\beta_2} \leq \frac{(\alpha_1)^2}{4\beta_1}$. This in turn implies that $\frac{(\alpha_1)^2}{4\beta_1} \geq \alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2$, and hence, $(\alpha_1)^2 \geq 4\beta_1 (\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2)$. Therefore, Expression (31) and Expression (32) lead to positive values. However, value of \bar{G}_1 provided by

Expression (32) leads to lower annual costs, therefore, we set $\bar{G}_1 = \frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1 (\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2)}}{2\beta_1}$, which also satisfies Expression (30).

We show above the feasibility of (\bar{Q}_1, \bar{G}_1) for the retailer's problem if the first investment option is adopted. Note that in this solution,

$\bar{Q}_1 = \bar{Q}_2$ and $\bar{G}_1 = \frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1 (\alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2)}}{2\beta_1}$. Now, assume that (\bar{Q}_1, \bar{G}_1) leads to greater annual costs. Then, due to the objective function under the cap policy, it must be that $G_2 < G_1$. Since $\alpha_1 G - \beta_1 G^2$ is strictly increasing over those values of G such that $G \leq \frac{\alpha_1}{2\beta_1}$, it follows that

$$\alpha_1 \bar{G}_1 - \beta_1 (\bar{G}_1)^2 > \alpha_1 \bar{G}_2 - \beta_1 (\bar{G}_2)^2.$$

As $\alpha_2 \leq \alpha_1$ and $\beta_2 \geq \beta_1$, we have

$$\alpha_1 \bar{G}_2 - \beta_1 (\bar{G}_2)^2 \geq \alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2.$$

The above two inequalities jointly imply that $\alpha_1 \bar{G}_1 - \beta_1 (\bar{G}_1)^2 > \alpha_2 \bar{G}_2 - \beta_2 (\bar{G}_2)^2$, which contradicts with Expression (29). Therefore, in contrary to our assumption, we must have $G_2 \geq G_1$. This implies the annual costs of (\bar{Q}_1, \bar{G}_1) along with the first investment option are lower than or equal to the optimum costs under the second investment option.

A.4 Proof of Theorem 2

Plugging $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2$ in place of $E(Q, G)$ in the objective function, it turns out be

$$\frac{(A + p\hat{A})D}{Q} + \frac{(h + \hat{h}p)Q}{2} + (c + \hat{c}p)D + G - \alpha pG + p\beta G^2.$$

The Hessian matrix corresponding to the above function is

$$\begin{pmatrix} \frac{2D(A + \hat{A}p)}{Q^3} & 0 \\ 0 & 2p\beta \end{pmatrix},$$

with a determinant $\frac{4(A + p\hat{A})Dp\beta}{Q^3}$, which is greater than zero. Combined with the fact that $\frac{2D(A + \hat{A}p)}{Q^3} > 0$, this result implies the objective function is jointly and strictly convex in Q and G , and hence, Q_2^* and G_2^* should satisfy the following system of equations:

$$\begin{aligned} \frac{\partial TC_2}{\partial Q}(Q_2^*, G_2^*) &= -\frac{(A + p\hat{A})D}{(Q_2^*)^2} + \frac{(h + \hat{h}p)}{2} = 0, \\ \frac{\partial TC_2}{\partial G}(Q_2^*, G_2^*) &= 1 - \alpha p + 2p\beta G_2^* = 0. \end{aligned}$$

Solving for Q_2^* and G_2^* in the above two expressions leads to the result in the theorem. \square

A.5 Proof of Lemma 4

Under a tax policy, if there is no investment opportunity to reduce carbon emissions, the retailer minimises the following function to find Q :

$$TC_2(Q, 0) = \frac{(A + p\hat{A})D}{Q} + \frac{(h + p\hat{h})Q}{2} + (c + p\hat{c})D.$$

$TC_2(Q, 0)$ is minimised at $Q_2^*(0) = \sqrt{\frac{2(A+p\hat{A})D}{(h+p\hat{h})}}$. In turn, the retailer's annual costs at $Q_2^*(0)$ are

$$TC_2(Q_2^*(0), 0) = \sqrt{2(A + p\hat{A})(h + p\hat{h})D} + (c + p\hat{c})D,$$

and his/her annual carbon emissions are

$$E(Q_2^*(0), 0) = \frac{\sqrt{D}[\hat{A}(h + p\hat{h}) + \hat{h}(A + p\hat{A})]}{\sqrt{2(A + p\hat{A})(h + p\hat{h})}} + \hat{c}D.$$

Expressions (10) and (11) are then utilised to compute the differences $E(Q_2^*(0), 0) - E(Q_2^*, G_2^*)$ and $TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*)$. □

A.6 Proof of Lemma 5

By definitions of $TC(Q, G)$ and $TC_2(Q, G)$, we know that $TC(Q, G) \leq TC_2(Q, G)$ as $E(Q, G) \geq 0$. It then follows that $TC(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ because $E(Q_2^*, G_2^*) > 0$, as noted in Corollary 2. Furthermore, we have $TC(Q^0, 0) < TC(Q_2^*, G_2^*)$ because $(Q^0, 0)$ minimises $TC(Q, G)$. Combining this with the fact that $TC(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ leads to $TC_2(Q_2^*, G_2^*) > TC(Q^0, 0)$.

Now, let us prove the second part of the lemma. We have from Theorem 2 and Assumption (A2) that $E(Q_2^*, G_2^*) < E(Q_2^*, 0)$. The remaining part of the proof will follow by showing that $E(Q_2^*, 0) < E(Q^0, 0)$ in case $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$, and that $E(Q_2^*, 0) = E(Q^0, 0)$, in case $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. Therefore, we will conclude that $E(Q_2^*, G_2^*) < E(Q^0, 0)$ in all cases.

If $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_2^* = Q^0$, which implies $E(Q_2^*, 0) = E(Q^0, 0)$. We will analyse the case of $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$ in two parts. First, suppose that $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. In this case, we have $Q^e < Q_2^* < Q^0$. This further leads to $E(Q_2^*, 0) < E(Q^0, 0)$ due to the strict convexity of $E(Q, 0)$ and the fact that Q^e is the unique minimiser of $E(Q, 0)$. Now, suppose that $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. In this case, we have $Q^e > Q_2^* > Q^0$. It again follows from the strict convexity of $E(Q, 0)$ and the definition of Q^e that we have $E(Q_2^*, 0) < E(Q^0, 0)$. □

A.7 Proof of Lemma 6

We will prove the different parts of the lemma in the following two cases.

Case 1 $\beta_2 \geq \beta_1, \alpha_2 \leq \alpha_1$

It follows from $\beta_2 \geq \beta_1$ that we have $\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_2 p - 1}{\sqrt{\beta_1}}$. Also, the fact that $\alpha_2 \leq \alpha_1$ leads to $\frac{\alpha_2 p - 1}{\sqrt{\beta_1}} \leq \frac{\alpha_1 p - 1}{\sqrt{\beta_1}}$. Combining these two results, we have $\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 p - 1}{\sqrt{\beta_1}}$, and hence $\frac{(\alpha_2 p - 1)^2}{4p\beta_2} \leq \frac{(\alpha_1 p - 1)^2}{4p\beta_1}$. Expression (11) and the fact that $\frac{(\alpha_2 p - 1)^2}{4p\beta_2} \leq \frac{(\alpha_1 p - 1)^2}{4p\beta_1}$ jointly imply that the annual costs under the first investment option is lower than or equal to the annual costs under the second investment option.

Now, let us compare the annual emissions under the two investment options. It follows from $\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 p - 1}{\sqrt{\beta_1}}$ that $\alpha_1 p \sqrt{\beta_2} - \sqrt{\beta_2} \geq \alpha_2 p \sqrt{\beta_1} - \sqrt{\beta_1}$. Because $\beta_2 \geq \beta_1$, we have $2\sqrt{\beta_2} \geq 2\sqrt{\beta_1}$. Combining this with $\alpha_1 p \sqrt{\beta_2} - \sqrt{\beta_2} \geq \alpha_2 p \sqrt{\beta_1} - \sqrt{\beta_1}$ leads to $\alpha_1 p \sqrt{\beta_2} + \sqrt{\beta_2} \geq \alpha_2 p \sqrt{\beta_1} + \sqrt{\beta_1}$, which in turn implies $\frac{\alpha_1 p + 1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p + 1}{\sqrt{\beta_2}}$. Since $\frac{\alpha_1 p - 1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p - 1}{\sqrt{\beta_2}}$ and $\frac{\alpha_1 p + 1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p + 1}{\sqrt{\beta_2}}$, it follows that $\frac{\alpha_1^2 p^2 - 1}{\beta_1} \geq \frac{\alpha_2^2 p^2 - 1}{\beta_2}$, or equivalently, $\frac{1 - \alpha_1^2 p^2}{\beta_1} \leq \frac{1 - \alpha_2^2 p^2}{\beta_2}$. This implies, due to Expression (10), that annual emissions under the first investment option are lower than or equal to annual emissions under the second investment option.

Case 2 $\beta_2 \geq \beta_1, \alpha_2 > \alpha_1$

If the second investment option leads to greater annual costs than the first one does, then Expression (11) implies that $\frac{(\alpha_2 p - 1)^2}{4p\beta_2} < \frac{(\alpha_1 p - 1)^2}{4p\beta_1}$, or equivalently, that $\alpha_2 p \sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1 p \sqrt{\beta_2} - \sqrt{\beta_2}$. Now, in contrary to the lemma, assume that the annual emissions level resulting from the second investment option is lower than or equal to that of the first investment option. In mathematical terms, assume that $\frac{1 - \alpha_2^2 p^2}{4p^2 \beta_2} \leq \frac{1 - \alpha_1^2 p^2}{4p^2 \beta_1}$, which is equivalent to

$$\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} \times \frac{\alpha_2 p + 1}{\sqrt{\beta_2}} \geq \frac{\alpha_1 p - 1}{\sqrt{\beta_1}} \times \frac{\alpha_1 p + 1}{\sqrt{\beta_1}}.$$

Due to $\frac{(\alpha_2 p - 1)^2}{4p\beta_2} < \frac{(\alpha_1 p - 1)^2}{4p\beta_1}$, we have $\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} < \frac{\alpha_1 p - 1}{\sqrt{\beta_1}}$. Therefore, in order for the above inequality to hold, we should have $\frac{\alpha_2 p + 1}{\sqrt{\beta_2}} > \frac{\alpha_1 p + 1}{\sqrt{\beta_1}}$, or equivalently, $\alpha_2 p \sqrt{\beta_1} + \sqrt{\beta_1} > \alpha_1 p \sqrt{\beta_2} + \sqrt{\beta_2}$. Since $\beta_2 \geq \beta_1$, this implies $\alpha_2 p \sqrt{\beta_1} - \sqrt{\beta_1} > \alpha_1 p \sqrt{\beta_2} - \sqrt{\beta_2}$, which contradicts $\alpha_2 p \sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1 p \sqrt{\beta_2} - \sqrt{\beta_2}$. Therefore, if the second investment option leads to greater annual costs than the first one, it must be that the annual emissions level resulting from the second investment is greater than that of the first investment.

If the second investment option leads to lower than or equal annual costs than the first one, the annual emission levels of the two investment options depend on the second term of Expression (10). If $\frac{1 - \alpha_2^2 p^2}{4p^2 \beta_2} < \frac{1 - \alpha_1^2 p^2}{4p^2 \beta_1}$, or equivalently, $\frac{1 - \alpha_2^2 p^2}{\beta_2} < \frac{1 - \alpha_1^2 p^2}{\beta_1}$, holds, then the second investment option is better in terms of the retailer's annual emissions; otherwise, the annual emissions level is greater than or equal to that of the first investment option. \square

A.8 Proof of Theorem 3

Plugging $C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D + \alpha G - \beta G^2$ in place of X , the objective function turns out be

$$\frac{(A + c_p \hat{A})D}{Q} + \frac{(h + \hat{h}c_p)Q}{2} + c_p \beta G^2 + (1 - \alpha c_p)G + (c + \hat{c}c_p)D - c_p C.$$

Following similar steps to those in the proof of Theorem 2 for checking the structural properties of $TC_2(Q, G)$, it can be shown that $TC_3(Q, G)$ is also jointly and strictly convex in Q and G , and hence, Q_3^* and G_3^* should satisfy the following system of equations:

$$\begin{aligned} \frac{\partial TC_3}{\partial Q}(Q_3^*, G_3^*) &= -\frac{(A + c_p \hat{A})D}{(Q_3^*)^2} + \frac{(h + \hat{h}c_p)}{2} = 0, \\ \frac{\partial TC_3}{\partial G}(Q_3^*, G_3^*) &= 1 - \alpha c_p + 2c_p \beta G_3^* = 0. \end{aligned}$$

Solving for Q_3^* and G_3^* in the above two expressions leads to the result in the theorem. \square

A.9 Proof of Lemma 7

Under a cap-and-trade policy, if there is no investment opportunity to reduce carbon emissions, the retailer minimises the following function to find Q :

$$TC_3(Q, 0) = \frac{(A + c_p \hat{A})D}{Q} + \frac{(h + c_p \hat{h})Q}{2} + (c + c_p \hat{c})D.$$

$TC_3(Q, 0)$ is minimised at $Q_3^*(0) = \sqrt{\frac{2(A + c_p \hat{A})D}{(h + c_p \hat{h})}}$. In turn, the retailer's annual costs at $Q_3^*(0)$ are

$$TC_3(Q_3^*(0), 0) = \sqrt{2(A + c_p \hat{A})(h + c_p \hat{h})}D + (c + c_p \hat{c})D,$$

and his/her annual carbon emissions are

$$E(Q_3^*(0), 0) = \frac{\sqrt{D}[\hat{A}(h + c_p \hat{h}) + \hat{h}(A + c_p \hat{A})]}{\sqrt{2(A + c_p \hat{A})(h + c_p \hat{h})}} + \hat{c}D.$$

Expressions (12) and (13) are then utilised to compute the differences $E(Q_3^*(0), 0) - E(Q_3^*, G_3^*)$ and $TC_3(Q_3^*(0), 0) - TC_3(Q_3^*, G_3^*)$. \square

A.10 Proof of Lemma 8

The first part of the lemma follows from a similar discussion to the proof of Lemma 5 and Assumption A(3). The second part follows from comparing Equation (13) to $TC(Q^0, 0)$. \square

A.11 Proof of Lemma 10

Consider a tax policy with parameter $p > 0$. Let $C = E(Q_2^*, G_2^*)$. Note that $C > 0$ because $E(Q_2^*, G_2^*) > 0$. It follows from the expressions for $TC_1(Q, G)$ and $TC_2(Q, G)$, and the fact that $E(Q_2^*, G_2^*) > 0$ and $p > 0$, that we have $TC_1(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$. Furthermore, as $C = E(Q_2^*, G_2^*)$, the optimal solution of the tax policy (i.e. (Q_2^*, G_2^*)), is also a feasible solution for the newly designed cap policy. Let (Q_1^*, G_1^*) be the retailer's optimal solution under the cap policy. It follows from this definition that $TC_1(Q_1^*, G_1^*) \leq TC_1(Q_2^*, G_2^*)$. Combining this with $TC_1(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ leads to $TC_1(Q_1^*, G_1^*) < TC_2(Q_2^*, G_2^*)$. Also, note that $E(Q_1^*, G_1^*) \leq C$, therefore, $E(Q_1^*, G_1^*) \leq E(Q_2^*, G_2^*)$. \square

For the second part of the proof, consider a cap policy with parameter $C > 0$. Suppose that a tax policy with parameter $p > 0$ can be found so that $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$ and $E(Q_2^*, G_2^*) \leq E(Q_1^*, G_1^*)$. By definition of (Q_1^*, G_1^*) , $E(Q_1^*, G_1^*) \leq C$, thus $E(Q_2^*, G_2^*) \leq C$ as well. This implies that (Q_2^*, G_2^*) is a feasible solution to the retailer's problem under the cap policy. Because (Q_1^*, G_1^*) is the optimal solution under the cap policy, it must be that $TC_1(Q_1^*, G_1^*) \leq TC_2(Q_2^*, G_2^*)$. This contradicts $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$; therefore, a tax policy with the assumed characteristics cannot be found. \square

A.12 Proof of Lemma 11

By definition of (Q_1^*, G_1^*) , we know that $E(Q_1^*, G_1^*) \leq C$. Since $X = C - E(Q_1^*, G_1^*) \geq 0$, it follows from the expressions for $TC_1(Q, G)$ and $TC_3(Q, G)$ that $TC_3(Q_1^*, G_1^*) \leq TC_1(Q_1^*, G_1^*)$ for $c_p > 0$. Combining this with the fact that $TC_3(Q_3^*, G_3^*) \leq TC_3(Q_1^*, G_1^*)$, we have $TC_3(Q_3^*, G_3^*) \leq TC_1(Q_1^*, G_1^*)$.

For the second part of the proof, let us consider Expression (2). This expression, independent of the emission regulation type, assumes a minimum value of $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$ when $Q = Q^e$ units are ordered and $G = \frac{\alpha}{2\beta}$ monetary units are invested. Therefore, $E(Q_1^*, G_1^*) \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$. Furthermore, at very large values of c_p , (Q_3^*, G_3^*) approaches $(Q^e, \frac{\alpha}{2\beta})$ and $E(Q_3^*, G_3^*)$ approaches $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$. Therefore, a large enough value of c_p can be chosen such that $E(Q_1^*, G_1^*) \geq E(Q_3^*, G_3^*)$. \square