Aircraft Rescheduling with Cruise Speed Control

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Airline operations are subject to frequent disruptions typically due to unexpected aircraft maintenance requirements and undesirable weather conditions. Recovery from a disruption often involves propagating delays in downstream flights and increasing cruise stage speed when possible in an effort to contain the delays. However, there is a critical trade-off between fuel consumption (and its adverse impact on air quality and greenhouse gas emissions) and cruise speed. Here we consider delays caused by such disruptions and propose a flight rescheduling model that includes adjusting cruise stage speed on a set of affected and unaffected flights as well as swapping aircraft optimally.

To the best of our knowledge, this is the first study in which the cruise speed is explicitly included as a decision variable into an airline recovery optimization model along with the environmental constraints and costs. The proposed model allows one to investigate the trade-off between flight delays and the cost of recovery. We show that the optimization approach leads to significant cost savings compared to the popular recovery method delay propagation.

Flight time controllability, nonlinear delay, fuel burn and CO_2 emission cost functions, and binary aircraft swapping decisions complicate the aircraft recovery problem significantly. In order to mitigate the computational difficulty we utilize the recent advances in conic mixed integer programming and propose a strengthened formulation so that the nonlinear mixed integer recovery optimization model can be solved efficiently. Our computational tests on realistic cases indicate that the proposed model may be used by operations controllers to manage disruptions in real time in an optimal manner instead of relying on ad-hoc heuristic approaches.

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1. Introduction

Although the airline industry has been very successful in implementing optimization tools for the planning and scheduling of resources, dealing with frequent disruptions in daily operations remains a significant challenge. According to a recent report by Schumer and Maloney (2008), flight delays are increasing in an alarming manner and consequently causing enormous negative economic impact. They attribute the majority of flight delays in 2007 to late-arriving aircraft and disruptions in the national aviation system. On the other hand, rising fuel costs and emerging CO_2 emissions restrictions impose additional challenges for speedy disruption recovery. In this paper we propose a novel rescheduling optimization model that explicitly accounts for delay as well as fuel burn and CO_2 emission costs by incorporating cruise speed control into airline disruption management.

Airline operations planning is done in a sequential manner (see e.g., Clausen et al. 2010). First, the flight schedule is determined based on forecasts of passenger demand and other relevant information. Then specific types of aircraft are assigned to individual flights in the schedule, and sequences of flights are generated for each fleet (these planning stages are called fleet assignment and aircraft routing, respectively). In the subsequent crew scheduling phase, flight crew and cabin crew are assigned to each flight based on the already determined aircraft rotations. However, on the day of operation, the planned aircraft schedules can become infeasible because of external disruptions or internal failures. As discussed in Barnhart (2009, p. 273), "on the side of airlines, decision support software for recovery is perhaps at the stage where planning software was 15 years ago. While research is active and hardware and data support have improved substantially, optimization based decision support tools for rapid recovery are still at an early stage of implementation at the major airlines. This represents a difficult, but crucial future challenge." A similar conclusion is reached by Rapajic (2009): "Despite airlines' tremendous efforts to streamline their operations to minimize controllable costs and improve flight punctuality, system inefficiencies are continuously on the increase. They inevitably lead to a higher number of operational disruptions, and consequently unforeseen losses."

For a recent survey on airline schedule recovery, we refer the reader to Ball et al. (2007). A general review on

disruption management, including airline operations, can be found in Yu and Qi (2004). As summarized in "Irregular Operations" by Barnhart (2009), when disruptions occur, airline operations controllers adjust scheduled operations by

1. delaying flight departures until aircraft and/or crews are ready;

2. canceling flights;

3. rerouting or swapping aircraft (i.e., reassigning aircraft among a subset of flights);

4. calling in new crews or reassigning existing crews;

5. postponing the departure times of flights to prevent connecting passengers from missing their connections; and

6. reaccommodating disrupted passengers.

Burke et al. (2010) use a simulation model to observe the impact of a randomly generated disruption on KLM airlines' schedule. Their recovery strategies include swapping aircraft, canceling flights, and accepting delays. Petersen et al. (2012) study an integrated airline recovery problem using a single-day horizon and propose a separate mixed integer mathematical model for the schedule, aircraft, crew, and passenger recovery problems. They utilize a Benders decomposition/column generation approach to achieve the coordination among these four mathematical models. They also propose a sequential recovery algorithm to handle larger problems. For a recent review on airline disruption management, we refer the reader to Clausen et al. (2010). The earlier studies on aircraft recovery do not consider the speed control. In air traffic flow management literature, Bertsimas et al. (2011) give an integer programming model for deciding on an optimum combination flow management actions, including ground holding, rerouting, and speed control. In their model, the speed control is achieved with a number of discrete time units an aircraft must spend in each sector. The associated fuel burn and CO_2 emissions cost of adjusting speed are not considered. Vela et al. (2010) propose local flight based heading and speed changes to deal with the air traffic conflict resolution problem while minimizing fuel costs. Sherali et al. (2006) report that the airline optimization models are quite sensitive to the fuel consumption and flight arrival delay costs.

Airline operations control centers prepare a flight plan for each flight of the aircraft that can be programmed into aircraft automation (Midkiff et al. 2009). The plan includes selecting a route by considering timing, fuel burn, and ride conditions for the flight. While preparing a flight plan, dispatchers also consider company priorities (e.g., minimum fuel trajectory versus minimum time trajectory). Currently, these priorities are quantified via a "cost index" parameter, which is the ratio of time-related costs to fuel-related costs and is a major driver of the flight plan optimization as minimum time and minimum fuel trajectories can be quite different (Airbus 1998). Cook et al. (2009) propose dynamic cost indexing, allowing airlines to compute and change the cost index during the flight.

Although in the airline industry there is a realization that the choice of cruise speed has a critical impact on the trade-off between reducing delays versus reducing fuel cost, Boeing (2007) cites that airlines do not take full advantage of this alternative, although a recent airline case study suggested a potential annual savings of $4 to $5 million, "with a negligible effect on the schedule" (Cook et al. 2009).

The current industry standard of cost indexing (CI) as outlined in Airbus (1998) and Boeing (2007) does not fully capture the flexibility of controllable flight times. In this approach, the cockpit crew of a delayed flight sets a number between 0 and 500 or 0 and 9999 depending on the aircraft model to assess the impact of the ratio of time-related costs to fuel-related costs based on the locally available information at that time. However, it may also be preferable to speed up an on-time flight to arrive early to the announced schedule to be able to swap the aircraft with another one so as to minimize the overall cost. Since the pilots cannot foresee the impact of their local solutions on the overall network, it is very difficult to assess and take full advantage of speed control and how it could be useful to decrease the delay propagations in the network. For example, acknowledging the drawbacks of using a predetermined CI value, Jeppesen Technology Services–Aviation Operations also recommends calculating a dynamic CI value for each flight based on departure time estimates (Altus 2010). By stating weaknesses in the practical application of cost index, Air Canada utilizes a city-pair cost index values as described in Saint-Martin and Wagner (2009). It also states that such an approach requires the corporate schedule to be adjusted for all fleets on all routes, and hence it needs a systemwide global optimization tool.

Although it is important to have a flight management system that can control the cost indices dynamically to deal with an unexpected event (such as weather changes or dealing with disruptions), the benefits of such a local speed adjustment tool will be limited without a global optimization. By using a mathematical model, we can evaluate possible trade-offs associated with speed control and/or swapping aircraft explicitly (two most popular recovery strategies in the airline industry Kohl et al. 2007), to find a global optimum for the overall network.

Moreover, with the application of new environmental regulations on fuel burn and greenhouse gas emissions considerations are becoming significantly more important for airlines. As of January 1, 2012, all airlines operating on EU airports are brought into the European Union's Emissions Trading System (ETS), joining more than 10,000 power and industrial plants that have been active in the scheme since 2005. Airlines incur additional costs for acquiring the required CO_2 permits in carbon markets. For example, Lowther et al. (2008) have shown that continuous descent arrival procedures could be used to minimize the required thrust during arrival and the approach to landing, thereby reducing noise, emissions, and fuel usage. There are a few commercial applications such as the Attila aircraft arrival management system by the ATH group (http://www.athgrp.com/) that provides required time of arrival recommendation to airlines

taking into account passenger connections, gate availability, and fuel consumption.

The major difficulty with including speed control into airline recovery is that the fuel burn and carbon emissions are nonlinear in cruise speed. Consequently, modeling the nonlinear cost function accurately and solving the resulting nonlinear mixed integer programming formulation in a reasonable amount of time are critical for successful implementation of such an approach in real time. To this end we utilize recent advances in conic quadratic mixed integer optimization and propose a strengthened formulation so that the proposed nonlinear mixed integer optimization model can be used by operations controllers to deal with disruptions in real time to find optimal solutions rather than relying on ad hoc heuristic approaches. To the best of our knowledge, we propose the first mathematical model that considers the speed control as well as its impact on fuel cost and air surface quality as a recovery strategy to deal with disruptions.

This paper is organized as follows. In §2, we first give a formal definition of the aircraft recovery problem with cruise speed control and then present a numerical example illustrating the benefit of flight time controllability in disruption management. In §3 we give a brief overview of conic integer programming and review a conic strengthening method utilized in the current paper. In §4 we present nonlinear mixed integer optimization models for a single aircraft type and multiple aircraft types, respectively. In §5, we describe the proposed conic strengthening of the model. In §6 we give three possible extensions that incorporate nonlinear delay functions and match-up rescheduling. We test the proposed mathematical model on a real-world data computationally in §7 and conclude with a few final remarks in §8.

2. Cruise Speed Control and Numerical Example

2.1. Modeling Cost of Cruise Stage Compression

A typical flight involves several stages: taxi-out, takeoff, climb, cruise, descent, final approach, landing, and taxi-in. Although the cruise stage is the most fuel efficient portion of the flight, most of the fuel is burned during this longest stage for a typical flight. There is also little room for planned compression in other stages because they are generally dictated by local traffic and safety considerations. Therefore, for modeling fuel and carbon emissions costs, we ignore the fuel consumption changes in other stages. Since one of the main contributions of this study is adjusting the cruise speed to compensate for the time losses due to a disruption, we need to consider the adverse effect of increasing cruise speed on fuel and carbon emissions costs. In this section, we will first discuss how we can quantify the impact of cruise speed on fuel and carbon emissions costs and then demonstrate it on a numerical example.

2.1.1. Fuel Cost. Estimating the fuel burn of an aircraft during a flight is a critical task. There has been a growing interest in fuel burn performance of aircraft. Aircraft manufacturers, consultants, aviation authorities, and scholars have published numerous papers and reports on this topic. A methodology that has been widely used in the literature is developed by the Base of Aircraft Data (BADA) project of EUROCONTROL, the air traffic management organization of Europe (EUROCONTROL 2009). In this paper, we adopt the cruise stage fuel flow model developed by BADA.

The fuel flow model of BADA is based on the total energy model as discussed in detail in Appendix EC.1 (available as supplemental material at http://dx.doi.org/10.1287/opre.2014.1279). Consequently, for a given mass and altitude, an aircraft's cruise stage fuel burn rate (kg/min) as a function of speed v (km/min) can be calculated as

$$f_{cr}(v) = c_1 v^3 + c_2 v^2 + \frac{c_3}{v} + \frac{c_4}{v^2}, \tag{1}$$

where coefficients $c_i > 0$, $i = 1, \ldots, 4$, are expressed in terms of aircraft specific drag and fuel consumption coefficients as well as mass of aircraft, air density at given altitude, and gravitational acceleration. These coefficients can be obtained from the BADA user manual for 399 aircraft types (EUROCONTROL 2012).

Given the fuel burn rate expression, we can formulate the total fuel consumption at cruise stage. Assuming that the distance flown at cruise stage is fixed d^{cr} and the duration of cruise stage is d^{cr}/v, we can formulate the total fuel consumption as below:

$$F(v) = \frac{d^{cr}}{v} \cdot f_{cr}(v) = d^{cr}\left(c_1 v^2 + c_2 v + \frac{c_3}{v^2} + \frac{c_4}{v^3}\right). \tag{2}$$

An aircraft is most fuel efficient at its maximum range cruise (MRC) speed. In other words, the fuel consumption function, $F(v)$, is minimized at MRC. Although from a fuel consumption perspective, it is ideal to fly at MRC speed, cost of time and scheduling considerations often dictate higher speeds. Note that $F(v)$ is a strictly convex and increasing for velocities higher than its minimizer MRC speed.
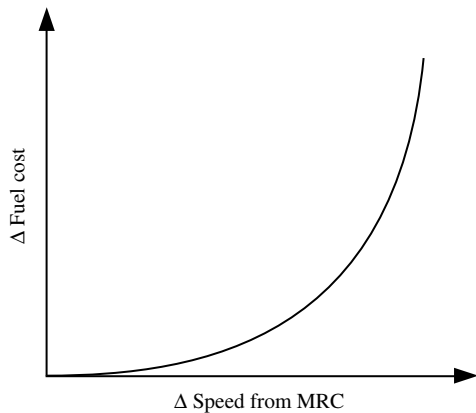
In the original schedule let the planned cruise speed be v^o, which might be greater than the MRC speed due to the labor and operating costs, and cruise time be t^o. Let v be the cruise speed variable; then we obtain the fuel cost change for the flight as

$$\Delta \text{Fuel Cost} = p_{\text{fuel}}(F(v) - F(v^o)), \tag{3}$$

where p_{fuel} is the price for jet fuel ($/kg). In Figure 1, we illustrate the percentage additional fuel cost as a function of speed increase from MRC speed as described in Boeing (2007).

Furthermore, one of the key factors in flight planning is determining the fuel load. Considerations in determining the

Figure 1. Additional fuel cost as a function of speed increase Δv.

Table 1. Schedules for aircraft N475AA and N554AA.

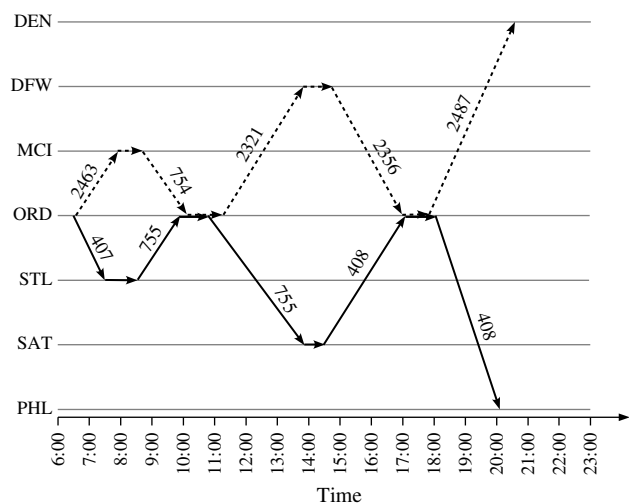| Tail no.	Flight no.	Departure airport	Arrival airport	Planned departure time	Planned flight time	Planned arrival time
N475AA	407	ORD	STL	6:20	1:10	7:30
	755	STL	ORD	8:35	1:15	9:50
	755	ORD	SAT	10:45	3:00	13:45
	408	SAT	ORD	14:30	2:40	17:10
	408	ORD	PHL	18:05	2:05	20:10
N554AA	2463	ORD	MCI	6:25	1:30	7:55
	754	MCI	ORD	8:40	1:30	10:10
	2321	ORD	DFW	11:15	2:35	13:50
	2356	DFW	ORD	14:40	2:20	17:00
	2487	ORD	DEN	17:50	2:45	20:35

2.2. A Numerical Example

We now give a numerical example to illustrate how cruise speed control and swapping aircraft can be utilized for rescheduling after a disruption. We consider two aircraft with schedules retrieved from the U.S. Department of Transportation Bureau of Transportation Statistics Airline On-Time Performance Data database (BTS 2010).

Table 1 shows the tail numbers and flight numbers along with the origin and destination airports and planned departure and arrival times in local ORD time for all flights in the schedule. Each aircraft starts its route from ORD early in the morning and finishes at different airports late in the evening. The flights with the same flight number are denoted as through flights, such as flights 755 and 408.

Figure 2 gives the time-space network representation of the original schedule for considered aircraft N554AA and N475AA along with their paths. The flight arcs originate from the departure airport and end at destination airport. Ground arcs span the time the aircraft spend on the ground. The figure also shows the planned departure and arrival times for each flight.

fuel load include fuel requirements to destination including reserves (which vary depending on the type of flight, e.g., over water); destination weather and alternatives; off-optimum speed or altitude requirements; and mechanical discrepancies of the aircraft. Therefore, there is an upper bound on the cruise speed due to various physical requirements such as fuel storage capacity, cabin pressure and noise constraints.

2.1.2. CO_2 Emission Cost. The principal greenhouse gas pollutant emitted from aviation is carbon dioxide, CO_2. Therefore, we need to assess the adverse impact of speed adjustments on the CO_2 emission of an aircraft during the cruise stage of a flight. There are several methods to estimate the carbon emission such as the advanced emission model developed by EUROCONTROL and the System for assessing Aviation's Global Emissions (SAGE) by the U.S. Federal Aviation Administration (FAA). In both models, the fuel burn calculation is based on the data stored in BADA as discussed earlier. The CO_2 emission is shown to be proportional to the fuel consumption and calculated by using Boeing Fuel Flow Method 2 (BFFM2) initially developed by the Boeing Company (DuBois and Paynter 2006). Furthermore, according to Boeing (2009), more than 20 pounds of CO_2 is emitted per U.S. gallon of fuel burned. According to EUROCONTROL (2001) and ICAO (2010), 3.15 kilograms of CO_2 is produced for every kilogram of fuel burn. Validation assessments conducted by Kim et al. (2007) have shown that SAGE can predict fuel burn to within 3% using data from 60,000 flights of a major U.S. airline and 20,000 flights of two major Japanese airlines. Thus, the CO_2 emission change during the cruise stage can be formulated as

$$\Delta \text{Carbon Emission Cost} = p_{CO_2} \cdot \kappa \cdot (F(v) - F(v^o)), \quad (4)$$

where p_{CO_2} is the cost of carbon emission ($/kg) and κ is CO_2 emission constant.

Figure 2. Time space network for aircraft N475AA and N554AA.

In this example, we assume that aircraft N475AA and N554AA are of the same type. Each aircraft has the same number of seats and all seats are occupied in each flight. Therefore, any swap between two aircraft does not cause spilled passengers. Using the formula given in (4), total fuel burn (in kgs) for the considered aircraft type and flight l is calculated as

$$F_l(v) = d_l^{cr}\left(0.01v^2 + 0.16v + \frac{0.74}{v^2} + \frac{2,200}{v^3}\right),$$

where d_l^{cr} is the distance flown at cruise stage of flight l.

We assume that each flight is planned with a cruise speed of 14 km/min in the original schedule. For each aircraft, we assume a minimum turnaround time of 30 minutes on the ground between landing and next departure. We also assume that for each flight, noncruise stages take 30 minutes. Assuming $p_{fuel} = 1$ \$/kg and $p_{CO_2} = 0.02$ \$/kg, the cruise stage fuel and carbon costs for the initial schedule are calculated in Table 2.

On the given schedule for two aircraft, let us assume that the departure of the second flight for N475AA is delayed for 90 minutes. In coping with the delay, one alternative is right-shifting all succeeding flights of N475AA, called the delay propagation. In delay propagation, the only way to reduce delays is utilizing the idle times on ground. Due to departure delay, actual arrival time of flight 755(1) will be 11:20. Then, the aircraft will be available for the next departure at 11:50, so the departure of flight 755(2) will be delayed by 65 minutes. Similarly, flights 408(1) and 408(2) will be delayed by 50 and 25 minutes, respectively. The initial delay on flight 755(1) propagates along the path of aircraft N475AA. The resulting schedule and corresponding delay costs for each flight is given in Table 3. The total delay cost for delay propagation is then \$9,125. In Figure 3, we give a time space network representation of the new schedule. In this figure, dotted arcs show the original schedule of flights. The figure shows that for four flights of N475AA, we experience arrival delays. Getting closer to the end of the day, delays become shorter, which is due to available idle times for N475AA in the original schedule.

An alternative way of repairing a disrupted schedule is compressing only the cruise speed of the aircraft, denoted as

Table 2. Cruise stage cost calculation for initial schedule.

Tail no.	Flight no.	Cruise time (min)	d^{cr} (km)	Fuel and carbon cost ($)
N475AA	407	40	560	2,979.7
	755	45	630	3,352.1
	755	150	2,100	11,173.8
	408	130	1,820	9,684.0
	408	95	1,330	7,076.8
N554AA	2463	60	840	4,469.5
	754	60	840	4,469.5
	2321	125	1,750	9,311.5
	2356	110	1,540	8,194.1
	2487	135	1,890	10,056.4

Table 3. Cost calculation for delay propagation (DP).

Tail no.	Flight no.	Arrival delay (min)	Unit delay cost ($/min)	Delay cost ($)
N475AA	755	90	30	2,700
	755	65	45	2,925
	408	50	50	2,500
	408	25	40	1,000
Total		230	—	9,125

CSC strategy, while minimizing the sum of fuel, carbon emission and delay costs simultaneously as discussed in detail in §4.2. Disabling swaps and solving the mathematical model for the given disruption, we achieved a new schedule which is given in Figure 4. The length of arrival delays, cruise time compression (Δt) and the speed changes (Δv) along with resulting costs are given in Table 4. The results show that the total delay cost is reduced to 4,770 from 9,125
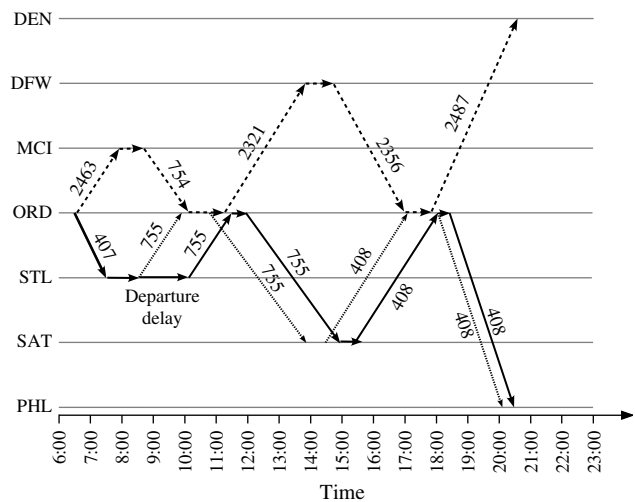
Figure 3. Time space network—after DP.
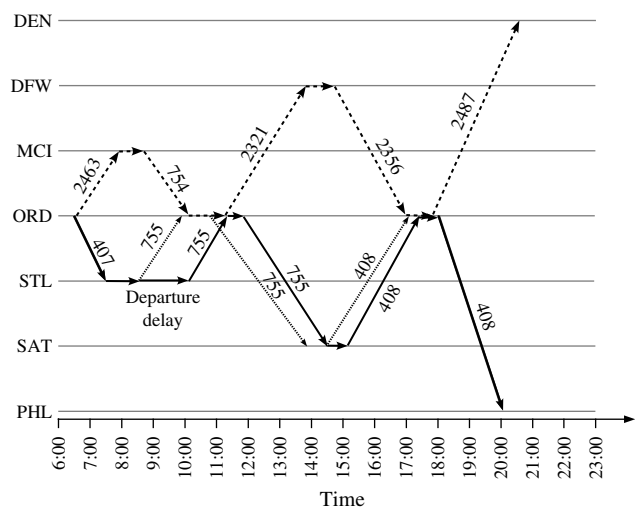

Figure 4. Time space network—after CSC.

Table 4. Cost calculation for CSC.

Tail no.	Flight no.	Arrival delay (min)	Δt	Δv (km/min)	Δ fuel cost	Δ carbon cost	Delay cost ($)	Total cost
N475AA	407	0	0	0	0	0	0	0
	755	84	6	2	412.3	26	2,520	2,958.3
	755	40	19	2	1,374.4	86.6	1,800	3,261
	408	9	16	2	1,191.1	75.0	450	1,716.2
	408	0	0	0	0	0	0	0
Total		133	—	—	2,977.8	187.6	4,770	7,935.5

Table 5. Cost calculation for S-CSC.

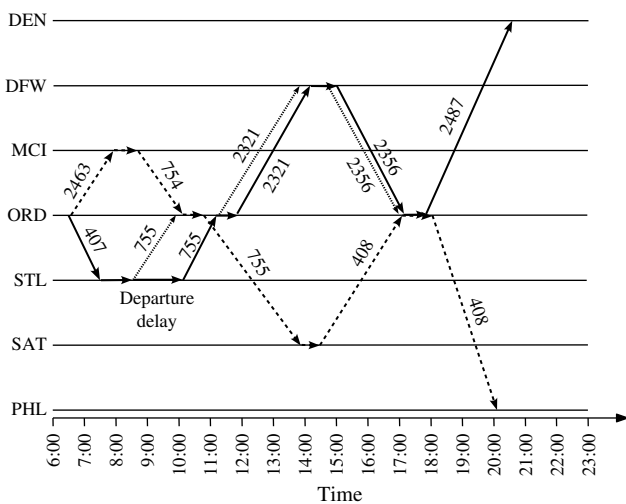| Tail no.	Flight no.	Arrival delay (min)	Δt	Δv	Δ fuel cost	Δ carbon cost	Delay cost ($)	Total cost
N475AA	407	0	0	0	0	0	0	0
	755	84	6	2	412.3	26	2,520	2,958.3
	2321	13	16	2	1,145.3	72.2	650	1,867.5
	2356	0	0	0	0	0	0	0
	2487	0	0	0	0	0	0	0
Total		97	—	—	1,557.6	98.2	3,170	4,825.8

by speeding up the aircraft N475AA on flights 755 and 408. As a result, additional fuel and CO_2 emission costs for N475AA are 2,977.9 and 187.6, respectively. In the new schedule, aircraft N475AA recovered from disruption and could catch the initial schedule at the departure time of its last flight. The recovery of the schedule was possible by adjusting speed and also utilizing idle times found in the schedule.

The final approach is considering aircraft swaps together with the cruise speed control, denoted as S-CSC strategy. In this strategy, two aircraft arriving at the same airport within a reasonably short period of time can be swapped. There may exist further restrictions on swaps like maintenance requirements and crew eligibility. In this example, aircraft can swap at ORD after completing flights 754 and 755(1), or 408 and 2356. The new schedule achieved by S-CSC is given in Figure 5. In an optimal solution of S-CSC model, two aircraft are swapped at ORD after completing flights

754 and 755(1). N475AA takes over flights 2321, 2356, and 2487. Flights 2321 experiences departure and arrival delays in this case, whereas flights 2356 and 2487 are on time. To reduce delay costs, the cruise speed is increased on flight 2321 assigned to N475AA in the optimal solution. On the other hand, N554AA operates flights 755 and 408 without any departure or arrival delays. Furthermore, these two swaps could be performed without incurring any crew deadhead cost. Cost calculations for S-CSC strategy is given in Table 5.

This example shows that when a disruption occurs, the cruise speed control can be an effective tool to decrease disruption costs. Here one should decide on the flights for which the cruise speed should be increased and the level of cruising speed. We also see that when rescheduling the flights, swapping can be used with cruise speed control. In this case, one should decide if there exists a feasible swap that can improve the overall cost. If there are alternative swaps, then which swap to select is a critical decision. Swap decisions also affect cruise speed decisions, as we see in the example.

3. Conic Integer Programming

In this section we give a brief overview of conic integer programming. Conic optimization refers to optimization of a linear function over conic inequalities. Since the late 1980s starting with Nesterov and Nemirovski (1988, 1990, 1991), convex conic optimization has experienced significant advances. The large number of practical applications and the availability of efficient algorithms make the conic quadratic (second-order conic) case particularly interesting. A conic quadratic constraint on $x \in \mathbf{R}^n$ is a constraint of the form

$$\|Ax - b\| \leqslant c'x - d.$$

Here $\|\ \|$ is the L_2 norm, A is an $m \times n$-matrix, b is an m-column vector, c is an n-column vector, and d is a scalar.

Figure 5. Time space network—after S-CSC.

We refer the reader to Lobo et al. (1998) and Alizadeh and Goldfarb (2003) for reviews on conic quadratic optimization and its applications.

Although there is an extensive body of literature of convex conic quadratic optimization, development in conic optimization with integer variables is quite recent. Çezik and Iyengar (2005) give linear and convex quadratic cuts for mixed 0-1 conic programs. Atamtürk and Narayanan (2008) introduce polymatroid inequalities to help solve special structured conic quadratic programs efficiently. Atamtürk and Narayanan (2010) give conic mixed integer rounding inequalities for conic quadratic mixed integer programs. Atamtürk and Narayanan (2011) propose lifting methods for conic mixed integer programming. With the availability of commercial solvers, conic integer models recently started to see use in applications in portfolio optimization (Vielma et al. 2008), scheduling (Aktürk et al. 2010), and supply chain problems (Atamtürk et al. 2012).

One of the common uses of conic quadratic inequalities is to represent a hyperbolic inequality

$$x_1^2 \leqslant x_2 x_3, \tag{5}$$

on x_1, x_2, $x_3 \geqslant 0$. It is easily verified that each hyperbolic inequality (5) can then be equivalently written as a conic quadratic inequality

$$\|(2x_1, x_2 - x_3)\| \leqslant x_2 + x_3. \tag{6}$$

In §5, we will present strong mixed 0-1 models using a series of hyperbolic inequalities.

4. Mathematical Formulations

In this section, we give the formal definition and mathematical formulation of the problem. For ease of readability we start with the simpler case for a single aircraft type and then extend the model for multiple aircraft types case.

4.1. Formulation for Single Aircraft Type

We first give a list of parameters and decision variables used in the model.

Parameters:

- L: set of flights in the schedule;
- L_d: set of initially disrupted flights;
- d_l: original departure time of flight l;
- r_l: original arrival time of flight l;
- a_l: turnaround time of flight l (min);
- w_l: arrival delay cost for flight l ($/min);
- h_{lj}: crew deadhead cost of swap between flights l and j;
- $S(l)$: set of flights that may be swapped with flight l;
- v_l^o: original cruise speed for the aircraft on flight l;
- v_l^u: maximum cruise speed of flight l;
- t_l^o: original cruise time for the aircraft on flight l;
- t_l^{nc}: noncruise time of flight time for the aircraft on flight l;

- ℓ_l^u: upper bound on the departure delay for flight l;
- $\bar{\ell}_l$: initial departure delay for flight $l \in L_d$;
- $n(l)$: immediate successor flight of flight l in the original schedule;
- κ: CO_2 emission constant (3.15);
- $Cmax$: overall upper bound on the carbon emission of all flights; and
- $F_l(v_l)$: cruise stage fuel burn as a function velocity for flight l.

Decision variables:

- v_l: cruise speed for aircraft on flight l (km/min);
- t_l: cruise time for aircraft on flight l (min);
- ℓ_l: departure delay on flight l (min); and
- x_{lj}: 1 if the aircraft of flight l and flight j are swapped at their destination and 0 otherwise.

Then with a departure delay of $\ell_l \geqslant 0$, the new arrival time of flight l with new cruise time t_l is $\tilde{r}_l = d_l + \ell_l + t_l^{nc} + t_l = r_l + \ell_l + t_l - t_l^o$; therefore, the delay incurred equals $\tilde{r}_l - r_l = \ell_l + t_l - t_l^o$.

Given a set of disrupted flights with certain departure delays, the model considers rescheduling all subsequent flights within a rescheduling horizon. The goal is to find new cruise speed levels and aircraft swaps to minimize total disruption cost including delay, fuel, CO_2 emission, and swap costs. In this model, departure delays are tracked in two ways. If a flight is a disrupted one, then the departure delay is known at the beginning and is fixed in the model, denoted as $\bar{\ell}_l$. Otherwise, a flight may experience a propagated delay. A propagated delay, if it exists, is calculated in the following way. For each flight, the model calculates the earliest time the assigned aircraft is ready for the departure of a flight. If this ready time is later than the planned departure time, then the flight experiences a departure delay. If the aircraft of flight l is not swapped, then the ready time for flight $n(l)$ is determined by actual arrival time of flight l and turnaround time. If the aircraft of flights l_1 and l_2 are swapped, then aircraft ready time for flight $n(l_1)$ ($n(l_2)$) is determined by l_2 (respectively, l_1).

Using the notation above, we now state the mathematical model of the problem:

$$\min \left\{ \sum_{l \in L} w_l \max\{\ell_l + t_l - t_l^o, 0\} + \sum_{l \in L} \sum_{j \in S(l)} h_{lj} x_{lj} \right.$$
$$\left. + \sum_{l \in L} (p_{\text{fuel}} + p_{CO_2} \kappa)(F_l(v_l) - F_l(v_l^o)) \right\} \tag{7}$$

$$\text{s.t.} \quad d_l^{\text{cr}} = v_l t_l, \quad l \in L, \tag{8}$$

$$0 \leqslant \ell_l \leqslant \ell_l^u, \quad l \in L, \tag{9}$$

$$\ell_l = \bar{\ell}_l, \quad l \in L_d, \tag{10}$$

$$v_l^o \leqslant v_l \leqslant v_l^u, \quad l \in L, \tag{11}$$

$$\sum_{l \in L} \kappa(F_l(v_l) - F_l(v_l^o)) \leqslant Cmax, \tag{12}$$

$$r_l + \ell_l + t_l - t_l^o + a_l \leqslant d_{n(l)} + \ell_{n(l)}$$
$$+ \sum_{j \in S(l)} \big[(r_l + \ell_l + t_l - t_l^o + a_l)$$
$$- (r_j + \ell_j + t_j - t_j^o + a_j) \big] x_{lj}, \quad l \in L, \quad (13)$$

$$\sum_{j \in S(l)} x_{lj} \leqslant 1, \quad l \in L, \quad (14)$$

$$x_{lj} = x_{jl}, \quad l \in L, \ j \in S(l), \quad (15)$$

$$x_{lj} \in \{0, 1\}, \quad l \in L, \ j \in S(l). \quad (16)$$

For a given initial schedule, our aim is to find a new schedule under a single (or multiple) disruption(s) to minimize the sum of four different cost components. The first term in the objective function is the sum of tardiness cost for all flights. If the departure delay cannot be recovered by flight time compression, then a tardiness cost is incurred. The second term in the objective function is the deadhead cost, which is incurred if two aircraft are swapped and if they end up at different airports than originally planned in the initial schedule. The third term in the objective function is the additional fuel and carbon emission cost due to increased cruise speed in the new schedule.

Constraint (8) sets the relationship between the cruise speed and the cruise time. We assume that the cruise flight distance is constant for a flight and, therefore, it is given by the product of initial cruise speed (v_l^o) and initial cruise time (t_l^o). Thus, the new cruise speed of an aircraft and the new cruise time are inversely proportional. Note that this is a nonlinear constraint.

Moreover, the departure times of the revised schedule cannot be earlier than the originally scheduled departure times and cannot be delayed longer than ℓ_l^u as stated in Constraint (9). Constraint (10) initializes departure delays for disrupted flights. Constraint (11) defines the cruise speed upper bound due to various physical requirements such as fuel storage capacity, cabin pressure, and noise constraints.

The air surface quality is measured in terms of the carbon emission as a function of the cruise speed. The environmental considerations are considered both in the objective function as well as in the constraints. Constraint (12) guarantees that the additional total carbon emission cannot exceed an upper bound *Cmax*.

In the initial schedule, there is a planned path for each aircraft. Constraint (13) ensures the precedence relations among the flights assigned to an aircraft are maintained in the new schedule. For flight l, if no aircraft swapping takes place, then constraint (13) ensures that

$$r_l + \ell_l + t_l - t_l^o + a_l \leqslant d_{n(l)} + \ell_{n(l)};$$

that is, the next flight $n(l)$ does not depart before the new arrival time of flight l plus its turnaround time. On the other hand, if aircraft of flights l and j are swapped after landing, then constraint ensures that

$$r_j + \ell_j + t_j - t_j^o + a_j \leqslant d_{n(l)} + \ell_{n(l)},$$

enabling incoming aircraft of flight j to catch flight $n(l)$. Note that in the example in Figure 5, we have arriving aircraft of flight 754 and flight 755(1) swapped. Swap decisions $x_{754, 755(1)} = x_{755(1), 754} = 1$ and constraints (13) for flights 754 and 755(1) ensure the precedence constraints on the aircraft paths after the swap.

Constraint (14) ensures that there can be at most one aircraft swap for each pair of flights. Constraint (15) guarantees the symmetry of swap decisions between flights. Finally, constraint (16) states that aircraft swapping decision variables x are binary.

Operations controllers decide which swaps are possible ($S(l)$) because certain conditions must hold for a swap. These conditions can be related to crew schedules, certifications of pilots to operate different aircraft, and maintenance requirements of aircraft.

An interesting feature of the presented model is that the problem is formulated without keeping track of individual aircraft, which simplifies the model substantially. Also note that the formulation is a mixed integer nonlinear optimization model with convex cost functions in the objective and nonlinear constraints (8), (12), and (13). However, using the reformulations described in the subsequent sections, we are able to solve relatively larger problems very efficiently.

4.2. Formulation for Multiple Aircraft Types

In this section, we generalize to the model for multiple types of aircraft. Although the ability to swap different types of aircraft introduces greater flexibility to the rescheduling problem, it comes with several challenges. First, because different aircraft types have different fuel burn and CO_2 emissions, flight delay and cruise speed decisions depend on the aircraft assignments. Second, swapping aircraft types with different number of seats can lead to spilling passengers on the smaller aircraft.

In order to model the multiple aircraft generalization, we first redefine some of the parameters and decision variables by adding an index for aircraft type:

Parameters:

 t_{lf}^o: cruise time of flight l for aircraft type f;
 t_{lf}^{nc}: noncruise time of flight l for aircraft type f;
 v_{lf}^o: cruise speed of flight l for aircraft type f;
 v_{lf}^u: maximum cruise speed of flight l for aircraft type f; and

$F_{lf}(v_{lf})$: fuel burn function for flight l and aircraft type f.

 Decision variables:

 t_{lf}: cruise time for flight l and aircraft type f;
 v_{lf}: cruise speed for flight l and aircraft type f; and
 ℓ_{lf}: departure delay on flight l for aircraft type f.

We also define the following new parameters and variables:

Parameters:

 F: set of aircraft types in the schedule;
 f_l: aircraft type of flight l in the original schedule;

psg_l: number of passengers on flight l in the original schedule;

b_f: number of seats on aircraft type f;

P: set of flight paths in the original schedule; and

$L(p)$: set of flights on path p.

Decision variables:

z_{lf}: 1 if aircraft type f is assigned to flight l and 0 otherwise; and

s_l: number of spilled passengers from flight l in the revised schedule.

For multiple aircraft case we redefine the fuel burn function as

$$F_{lf}(v_{lf}) = \begin{cases} d_l^{\mathrm{cr}}\left(c_1 v_{lf}^2 + c_2 v_{lf} + \dfrac{c_3}{v_{lf}^2} + \dfrac{c_4}{v_{lf}^3}\right), & \text{if } z_{lf} = 1, \\ 0, & \text{if } z_{lf} = 0; \end{cases}$$

(17)

so that if aircraft f is not assigned to flight l, then $F_{lf}(v_{lf}) = 0$.

Now we describe the constraints used for modeling the multi-aircraft case. The first constraint assigns a single aircraft type to each flight:

$$\sum_{f \in F} z_{lf} = 1, \quad l \in L.$$

For each aircraft route $p \in P$, let $l_o(p)$ be the first flight of the route considered in rescheduling. For these flights we set the aircraft type assignments according to the assignments in the original schedule:

$$z_{l_o(p)f_{l_o(p)}} = 1, \quad p \in P.$$

The following sets of constraints relate aircraft swap decisions to aircraft type assignments. If aircraft of flight l is not swapped with another after landing, then aircraft type assignments of flights l and its immediate successor $n(l)$ will be the same. If no aircraft swap occurs immediately after l, then $x_{lj} = 0$ for all possible swaps $(S(l))$. Thus, for each aircraft type f, $z_{n(l),f} = z_{l,f}$. That is,

$$|z_{n(l),f} - z_{lf}| \leqslant \sum_{j \in S(l)} x_{lj}, \quad l \in L, f \in F.$$

On the other hand, if an aircraft swap occurs between flights l and j after landing, i.e., $x_{lj} = 1$, then corresponding aircraft type assignments will apply for the immediate successors of l and j. That is, if aircraft of l and j are swapped, then the successor of l ($n(l)$) is taken over by the aircraft of j in the original schedule, i.e., aircraft type f_j, modeled as

$$z_{n(j)f_l} \geqslant x_{lj}, \quad l \in L, j \in S(l),$$

$$z_{n(l)f_j} \geqslant x_{lj}, \quad l \in L, j \in S(l).$$

In the multiple aircraft case we include a new constraint, which limits the number of swaps on a flight route or aircraft:

$$\sum_{l \in L(p)} \sum_{j \in S(l)} x_{lj} \leqslant 1, \quad p \in P.$$

When modeling the departure and delay times of flight $n(l)$ we need to calculate the ready time of the assigned aircraft. Ready time of the aircraft of flight $n(l)$ is the arrival time of the aircraft's previous flight and the turnaround time. The aircraft assigned to a flight is determined by the previous swaps made on the path. Two cases can occur:

1. No swap is made on the path of the initially assigned aircraft before flight l.

2. A swap has occurred before flight l.

In the first case, two scenarios may occur. The first one is that no swap is made after flight l. In this scenario, flight $n(l)$ is performed by its initially assigned aircraft. In the second scenario, flight $n(l)$ is performed by one of the aircraft ($f \in \{f_j: j \in S(l)\}$) swapped with f_l. Therefore, the ready time of the newly assigned aircraft has to be considered. These two scenarios in the first case are handled by the following constraint set for each $l \in L$:

$$d_{n(l)} z_{n(l)f_k} + \ell_{n(l)f_k} + (r_k + \ell_k^u + a_k)(1 - z_{n(l)f_k})$$
$$\geqslant d_k z_{kf_k} + a_k z_{kf_k} + \ell_{kf_k} + t_{kf_k} + t_{kf_k}^{nc} z_{kf_k},$$
$$k \in \{\{l\} \cup S(l)\}. \quad (18)$$

In the second case, a swap has occurred before flight l, so l is not assigned to its initial aircraft. Taking the single swap restriction for each aircraft, we can conclude that no swap will be made on this path after flight l. Thus, we only need to consider the ready time of the new aircraft of flight l after completing flight l and its turnaround time. This case is considered in the following constraint set for each $l \in L$:

$$d_{n(l)} z_{n(l)f} + \ell_{n(l)f} \geqslant d_l z_{lf} + a_l z_{lf} + \ell_{lf} + t_{lf} + t_{lf}^{nc} z_{lf},$$
$$f \in F \setminus \{f_l\}. \quad (19)$$

In the case of aircraft types with different number of seats, swaps may cause spilled passengers. In order to capture the cost for spilled passengers, we introduce to the model a new decision variable s_l to denote the number of spilled passengers due to swapping different aircraft types in the revised schedule. If a certain flight is assigned to a smaller aircraft (i.e., fewer number of seats than the originally scheduled aircraft), then some of the passengers already assigned to subsequent flights will be spilled. We introduce a penalty cost as a function of the number of spilled passengers. Let the parameter sp_l be the cost of each spilled passenger of flight l and the decision variable s_l be the number of spilled passengers in the revised schedule. The following constraints define the number of spilled passengers s_l on each flight $l \in L$:

$$psg_l - \sum_{f \in F} b_f z_{lf} \geqslant s_l \geqslant 0, \quad l \in L.$$

The complete formulation for multiple aircraft airline rescheduling problem is given below:

$$\min\left\{\sum_{l\in L}w_l\max\left\{\left[d_l-r_l+\sum_{f\in F}(\ell_{lf}+t_{lf}+t_{lf}^{nc}z_{lf})\right],0\right\}\right.$$

$$+(p_{\text{fuel}}+p_{CO_2}\kappa)\left(\sum_{l\in L,f\in F}F_{lf}(v_{lf})-\sum_{l\in L}F_{lf_l}(v_{lf_l}^o)\right)$$

$$\left.+\sum_{l\in L}\sum_{j\in S(l)}h_{lj}x_{lj}+\sum_{l\in L}sp_ls_l\right\}\qquad(20)$$

s.t.

$$d_l^{\text{cr}}z_{lf}=v_{lf}t_{lf},\quad l\in L,\ f\in F;\qquad(21)$$

$$0\leqslant\ell_{lf}\leqslant\ell_l^u z_{lf},\quad l\in L,\ f\in F;\qquad(22)$$

$$\ell_l=\bar{\ell}_l,\quad l\in L_d;\qquad(23)$$

$$v_{lf}^o z_{lf}\leqslant v_{lf}\leqslant v_{lf}^u z_{lf},\quad l\in L,\ f\in F;\qquad(24)$$

$$v_{lf},t_{lf}\geqslant 0,\quad l\in L,\ f\in F;\qquad(25)$$

$$\sum_{l\in L}\sum_{f\in F}\kappa F_{lf}(v_{lf})-\sum_{l\in L}\kappa F_{lf_l}(v_{lf_l}^o)\leqslant Cmax;\qquad(26)$$

$$d_{n(l)}z_{n(l)f_k}+\ell_{n(l)f_k}+(r_k+\ell_k^u+a_k)(1-z_{n(l)f_k})$$
$$\geqslant d_k z_{kf_k}+a_k z_{kf_k}+\ell_{kf_k}+t_{kf_k}+t_{kf_k}^{nc}z_{kf_k},$$
$$l\in L\ k\in\{\{l\}\cup S(l)\};\qquad(27)$$

$$d_{n(l)}z_{n(l)f}+\ell_{n(l)f}\geqslant d_l z_{lf}+a_l z_{lf}+\ell_{lf}+t_{lf}+t_{lf}^{nc}z_{lf},$$
$$l\in L,\ f\in F\setminus\{f_l\};\qquad(28)$$

$$\sum_{l\in L(p)}\sum_{j\in S(l)}x_{lj}\leqslant 1,\quad p\in P;\qquad(29)$$

$$\sum_{j\in S(l)}x_{lj}\leqslant 1,\quad l\in L;\qquad(30)$$

$$x_{lj}=x_{jl},\quad l\in L,\ j\in S(l);\qquad(31)$$

$$psg_l-\sum_{f\in F}b_f z_{lf}\geqslant s_l,\quad l\in L;\qquad(32)$$

$$\sum_{f\in F}z_{lf}=1,\quad l\in L;\qquad(33)$$

$$z_{l_o(p)f_{l_o(p)}}=1,\quad p\in P;\qquad(34)$$

$$|z_{n(l),f}-z_{lf}|\leqslant\sum_{j\in S(l)}x_{lj},\quad l\in L,\ f\in F;\qquad(35)$$

$$z_{n(j)f_l}\geqslant x_{lj},\quad l\in L,\ j\in S(l);\qquad(36)$$

$$z_{n(l)f_j}\geqslant x_{lj},\quad l\in L,\ j\in S(l);\qquad(37)$$

$$x_{lj}\in\{0,1\},\quad l\in L,\ j\in S(l);\qquad(38)$$

$$z_{lf}\in\{0,1\},\quad l\in L,\ f\in F;\qquad(39)$$

$$s_l\geqslant 0,\quad l\in L.\qquad(40)$$

Mathematical formulation given above is a mixed integer nonlinear optimization problem, which includes the nonlinear relation between cruise time and speed in constraint (21)

and the nonlinear and discontinuous fuel burn function (F) in the objective and in constraint (26). In the next section, we show that the nonlinear constraints in the mathematical model can be strengthened and reformulated using conic quadratic inequalities.

5. Strengthened Conic Quadratic Mixed Integer Model

One of the most critical aspects of disruption management is to be able to recover fast from a disrupted schedule. Nonlinear mixed integer optimization often requires very long computation time to come up with optimal or near-optimal solutions. In order to reduce the solution times, in this section we show how to strengthen and reformulate the preceding models. We take the conic quadratic reformulation approach described in Aktürk et al. (2009) and generalized in Günlük and Linderoth (2010). As demonstrated in §7, the proposed reformulations can be solved in reasonable time, within a few minutes for practical size problems. For simplicity of presentation, we drop the indices of the variables.

The fuel burn function

$$F(v)=\begin{cases}d_l^{\text{cr}}\left(c_1 v^2+c_2 v+\dfrac{c_3}{v^2}+\dfrac{c_4}{v^3}\right),&\text{if }z=1;\\0,&\text{if }z=0;\end{cases}$$

is discontinuous, and therefore its epigraph $E_F=\{(v,t)\in\mathbf{R}^2:F(v)\leqslant t\}$ is nonconvex. The next proposition describes how to convexify E_F. For more detail on this topic, we refer the reader to Aktürk et al. (2009) and Günlük and Linderoth (2010).

PROPOSITION 1. *The convex hull of E_F can be expressed as*

$$t\geqslant d_l^{\text{cr}}(c_1\tau_1+c_2 v+c_3\tau_3+c_4\tau_4),\qquad(41)$$

$$v^2\leqslant\tau_1 z,\qquad(42a)$$

$$z^4\leqslant\tau_3 v^2 z,\qquad(42b)$$

$$z^4\leqslant\tau_4 v^3,\qquad(42c)$$

in the constraint set. Furthermore, inequalities (42a)–(42c) *can be represented by conic quadratic inequalities.*

Perspective of a convex function $f(v)$ is $zf(v/z)$ (Hiriart-Urruty and Lemaréchal 2001). Since each of the nonlinear terms v^2, $1/v^2$, and $1/v^3$ in $F(v)$ is a convex function, epigraph of the perspective of each term can then be stated as

$$\frac{v^2}{z}\leqslant\tau_1,\qquad\frac{z^3}{v^2}\leqslant\tau_3,\qquad\frac{z^4}{v^3}\leqslant\tau_4,$$

respectively. Since $z,v\geqslant 0$, they can be rewritten as in the statement of the proposition.

Finally, observe that (42a) is a hyperbolic inequality, (42b) can be restated as two hyperbolic inequalities

$$z^2\leqslant wv\quad\text{and}\quad w^2\leqslant\tau_3 z,$$

and (42c) can be restated as

$$z^2 \leqslant wv \quad \text{and} \quad w^2 \leqslant \tau_3 v,$$

which can be written in conic quadratic inequality as described in §3.

Equality (21) guarantees that if the cruise stage speed of an aircraft on a flight is increased, then the cruise stage time is decreased appropriately. Equality (21) defines a curve and hence a nonconvex set of feasible points. Proposition 2 states that this constraint can be relaxed to a convex inequality and furthermore shows that the inequality can be restated as a conic quadratic inequality.

PROPOSITION 2. *For every optimal solution to* (20)–(40) *inequality,*

$$d^{\mathrm{cr}} z \leqslant vt \tag{44}$$

is satisfied as an equality. Moreover, inequality (44) *can be equivalently represented with the conic quadratic inequality.*

If $z = 1$, then for any fixed value of t because the objective function (7) is increasing in v for $v \geqslant v^o$, $v = d^{\mathrm{cr}}/t$ holds. In the other hand, if $z = 0$, then $v = 0$ because of constraint (24), and the equality holds again. Furthermore, using that z is a 0-1 decision variable and $v, t \geqslant 0$, inequality (44) can be equivalently written as a hyperbolic inequality

$$d^{\mathrm{cr}} z^2 \leqslant vt,$$

which can be stated as a conic quadratic inequality as described in §3.

6. Extensions to the Model

In this section, we present three extensions to the model given in §4.2. The first extension is nonlinear delay costs in §6.1. In §6.2 we present step delay function form of delay cost. Finally, in §6.3 we present a matchup formulation.

6.1. Nonlinear Flight Delay Cost

In the models presented in §4, we use a linear penalty for the arrival tardiness of each flight as is common in the airline recovery literature. However, Hoffman and Ball (2000) in their ground holding model suggest using a nonlinear delay cost function would be more attractive since flight delay costs tend to grow with time at a greater rate than linear rate. Moreover, Hansen et al. (2001) perform a detailed investigation of airline cost functions and also report that the cost of a delay varies nonlinearly with the duration of the delay. Therefore, we could easily replace the linear delay cost in our objective function with a nonlinear one such that the delay cost can be penalized using a convex increasing function of tardiness t:

$$g(t) = wt^\sigma, \tag{45}$$

where $w > 0$ and $\sigma > 1$. Aktürk et al. (2009) describe how to strengthen the epigraph of such a function and represent it using conic quadratic inequalities.

Figure 6. Delay cost as a step function of delay.
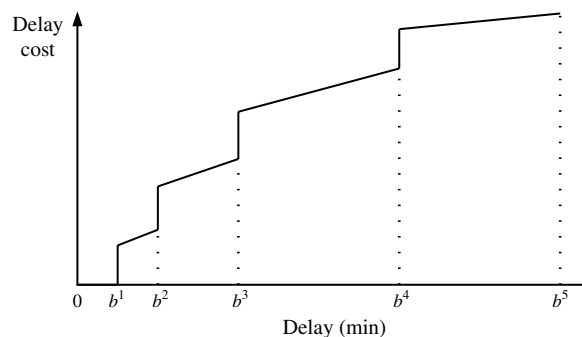
6.2. Step Function Form Delay Costs

An alternative way of modeling delay cost is representing it as a step function. Figure 6 gives a typical situation where cost increases as a function of delay in discrete steps. The breakpoints could correspond to the cases where a flight is considered "delayed" if it is b^1 minutes later than its scheduled time. For example, a tardiness of b^2 minutes or more could cause missing baggage connections, whereas passengers could miss their connecting flights if tardiness is above b^3 minutes. Above a certain tardiness value, the aircraft can miss its next flight.

Delay cost in the form of a step function can be incorporated into our multiple aircraft airline rescheduling model in the following way. We define a decision variable td_l^m, the amount of arrival delay for flight l if mth segment is active in delay cost function. We introduce a set of 0-1 decision variables y_l^m that indicate if the amount of delay is in the mth segment of the step function or not. If the mth segment is active, then the delay amount variable td_l^m can take positive values. Also, we define a decision variable td_l for the amount of delay on flight l. Let w_l^m be the unit delay cost, and c_l^m be the fixed cost if delay amount t_l^d is in the mth interval. Assume that flight l has m_l segments in its delay function. Then for each $l \in L$, we replace the delay cost term in the objective of the model with the term below:

$$\sum_{l \in L} \sum_{m=1}^{m_l} \left\{ c_l^m y_l^m + w_l^m td_l^m \right\}.$$

Furthermore, we add the following constraints

$$b_l^{m-1} y_l^m \leqslant td_l^m \leqslant b_l^m y_l^m \quad m = 1, \ldots, m_l, \quad \forall l \in L, \tag{46}$$

$$d_l - r_l + \sum_{f \in F} (\ell_{lf} + t_{lf} + t_{lf}^{nc}) \leqslant td_l \quad \forall l \in L, \tag{47}$$

$$\sum_{m=1}^{m_l} td_l^m = td_l, \quad \forall l \in L, \tag{48}$$

$$y_l^m \in \{0, 1\} \quad m = 1, \ldots, m_l, \quad \forall l \in L, \tag{49}$$

Constraint (46) sets the bounds for td_l^m so that if $y_l^m = 1$, td_l^m takes a delay in the segment $[b_l^{m-1}, b_l^m]$. Constraint (47) determines the length of delay if exists. Constraint (48) relates the delay to the segments of the step function.

6.3. Match-Up Model

Finally we consider the case where two swapped aircraft are required to swap again later so that each aircraft comes back to its original route at some point before the end of scheduling horizon. We start with defining set $L(f)$ that is the set of flights assigned to aircraft f in the original schedule. Then, we introduce a decision variable $y_{f_1 f_2}$ which is 1, if swaps occur between the paths of aircraft f_1 and f_2 and 0 otherwise.

In order to model the multiple aircraft airline match-up rescheduling, we

1. replace constraints (27) and (28) with the constraint:

$$d_{n(l)} z_{n(l)f} + \ell_{n(l)f} + (r_k + \ell_k^u + a_k)(1 - z_{n(l)f})$$
$$\geqslant d_k z_{kf} + a_k z_{kf} + \ell_{kf} + t_{kf} + t_{kf}^{nc} z_{kf},$$
$$l \in L, \ f \in F, \ k \in \{\{l\} \cup S(l)\}; \quad (50)$$

2. remove the single swap constraint (29);

3. add the constraint that between two aircraft, either two or no swaps occur:

$$\sum_{l \in L(f_1)} \sum_{j \in \{L(f_2) \cap S(l)\}} x_{lj} = 2 y_{f_1 f_2}, \quad f_1, f_2 \in F, \ f_1 \neq f_2; \quad (51)$$

4. add the constraint so that an aircraft f_1 cannot be swapped with more than one aircraft:

$$\sum_{f_1 \neq f_2} y_{f_1 f_2} \leqslant 1, \quad f_1 \in F; \quad (52)$$

5. and replace constraints (36) and (37) with the constraints below:

$$x_{lj} \leqslant 1 - z_{lf_l} + z_{n(l)f_j}, \quad l \in L, \ j \in S(l); \quad (53)$$
$$x_{lj} \leqslant 1 - z_{lf_j} + z_{n(l)f_l}, \quad l \in L, \ j \in S(l). \quad (54)$$

Constraint (53) guarantees that if there is a swap after flights l and j, and flight l is assigned to f_l, i.e., $z_{lf_l} = 1$, then flight $n(l)$ must be assigned to aircraft f_j. Similarly, constraint (54) handles the case that l is assigned to f_j.

In the next section, we present our computational results on the proposed rescheduling model.

7. Computational Results

In our computational study, we test three different approaches for repairing a disrupted airline schedule. The first one is the delay propagation (DP), or push-back recovery, as discussed in Schaefer and Nemhauser (2006), in which a delayed flight causes delays on subsequent flights of the disrupted aircraft and the only time saving option available is to utilize the excess planned ground time. The second approach is cruise speed control (CSC), in which cruise speed of aircraft is adjusted in order to trade off fuel (and carbon emission) costs and delay cost. Finally, the third approach is swap and cruise speed control (S-CSC), which allows

swapping aircraft between flights in addition to adjusting cruise speed. Our overall goal is to minimize the sum of fuel, carbon emission, delay, and swap costs. Jet fuel and carbon emission prices could be obtained from different sources, but estimating the delay cost is more difficult. Therefore, the delay cost is one of the experimental factors in our computational study and is set to low as well as high levels in the experiments. Furthermore, we have studied linear, nonlinear, and stepwise delay costs separately. Our computational studies have shown that the higher the impact of the delay cost, the more valuable speed control is as a recovery strategy.

We test all three approaches on two different flight schedules extracted from the database "Airline On-Time Performance Data," provided by the Bureau of Transportation Statistics of the U.S. Department of Transportation (BTS 2010). To construct the first schedule, called the ORD schedule, from the database we queried the planned departure and arrival times of all American Airlines (AA) flights for the date of 01/27/2010. Then we filtered the schedules of aircraft that first departed from Chicago O'Hare International Airport (ORD) and revisited ORD at least once on the same day. This allowed us to study the schedules of aircraft for which ORD may be considered as a base airport. The schedule ORD has 114 flights operated by 31 different aircraft. The second schedule, called the DFW schedule, is constructed the same way for Dallas-Fort Worth International Airport. DFW has 207 flights and 60 aircraft, almost double the size of ORD schedule in terms of the number of flights and the number of aircraft. We give the details of ORD and DFW schedules in the e-companion to this paper. ORD and DFW are two highly congested airports in the United States with a large number of flights.

For the computational study, we adopt the fuel burn model of BADA (EUROCONTROL 2009) summarized in 2.1.1. We consider six different types of aircraft and randomly assign each aircraft to an aircraft type among the alternatives given in Table 6. The fuel burn related parameters are from operations performance files provided by BADA (EUROCONTROL 2012). Assuming that an aircraft's mass is fixed at an average value and a fixed altitude through the cruise stage, we have generated fuel burn parameters $(c_i, i = 1, 2, 3, 4)$ for each aircraft type. We assume that gravitational acceleration is 9.80665, atmospheric density at cruise stage altitude is 0.38 kilograms per cubic meter, and there are 0 degrees of block angle at cruise stage. For each aircraft type, we list fuel burn related parameters, corresponding MRC speed levels, and number of seats in Table 6.

For ORD and DFW schedules retrieved from the BTS database, for each flight, we have planned departure, planned arrival, and planned flight times. As an example, Table 7 gives the schedule of aircraft N530AA on 01/27/2010 by local time in ORD airport. In our experiments we assume that all noncruise stages of a flight, such as taxiing, take-off, climb, and descent, take 30 minutes in total. Flight 398 given

Table 6. Parameters for six aircraft types.

Parameters	Aircraft types					
	MD83	B727 228	B737 500	B767 200ER	A320 212	A320 111
Mass (m, kgs)	61,200	74,000	50,000	140,000	64,000	62,000
Reference wing area (S, m^2)	118	157.9	105.4	283.3	122.6	122.4
Drag coefficient ($C_{D0,CR}$)	0.021154	0.018	0.018	0.014	0.024	0.024
Drag coefficient ($C_{D2,CR}$)	0.046897	0.06	0.055	0.049	0.0375	0.0375
1st thrust specific fuel consumption coefficient (C_{f_1}, kg/min·kN)	0.74627	0.53178	0.46	0.338	0.94	0.94
2nd thrust specific fuel consumption coefficient (C_{f_2}, knots)	638.59	276.72	300	192	100,000	50,000
Cruise fuel flow factor (C_{fcr})	0.95057	0.954	1.079	0.9876	1.06	1.095
c_1	0.002439115	0.004583469	0.002761029	0.006065562	0.00002579	0.00005319
c_2	0.093455678	0.076100255	0.049698524	0.069875269	0.154734277	0.159582672
c_3	50.21840018	115.8540941	65.54794936	178.995095	0.37911718	0.736282025
c_4	1,924.13809	1,923.548696	1,179.863088	2,062.023495	2,274.703078	2,208.846074
MRC (km/min)	14.46	14.46	14.32	14.66	14.48	14.25
Number of seats	148	134	122	181	180	172

Table 7. Flight schedule of N530AA on 01/27/2010.

Flight no	Departure airport	Destination airport	Planned departure time	Planned flight time	Planned arrival time
398	ORD	LGA	6:15 A.M.	135	8:30 A.M.
319	LGA	ORD	9:25 A.M.	170	12:15 P.M.
2329	ORD	DFW	1:35 P.M.	155	4:10 P.M.
2364	DFW	ORD	5:00 P.M.	150	7:30 P.M.

Table 8. Experimental parameters.

Parameter	Value
Number of delayed aircraft (N_d)	1; 2
Length of departure delays (L_d)	Low: $U[45, 75]$; high: $U[90, 120]$
Delay cost level (w_l)	Low: $U[10, 30]$; high: $U[50, 100]$ ($/min)
Spilled passenger cost (sp_l)	$U[50, 100]$ ($/pax)
Maximum allowable departure delay (ℓ_l^u)	180 min
Upper bound on cruise speed (v_{lf}^u)	$1.1 \times v_{lf}^o$
Lower bound on cruise speed (v_{lf}^l)	v_{lf}^o
Turnaround time (a_l)	30 min or 45 min, depending on arrival airport
Jet fuel price (p_{fuel})	1 ($/kg)
Carbon emission price (p_{CO_2})	20 ($/ton)

in Table 7 is planned to depart from ORD at 6:15 A.M. and arrive at LGA at 8:30 A.M. (by local time in ORD). Planned flight time for flight 398 is 135 minutes, 105 minutes of which is assumed to be the cruise time in our experiments.

Considering ORD and DFW schedules, we generate random departure delays (in minutes) in the initial schedules. We apply a departure delay to the second flight of randomly selected aircraft. We randomly generate lengths of delays and delay costs. In our computational study we assume that in the original schedule each aircraft cruises at $1.02 \times$ MRC speed and we calculate the fuel consumption and carbon emission occurred in the original schedule accordingly. As stated in Delgado and Prats (2009), the cruise speed can be varied from maximum range cruise speed by about 10%. Table 8 lists the values of the parameters used in the experiments. In addition to those parameters given in the table, we calculate the location-dependent deadhead cost for the crews based on the flight times and fuel costs between the airports. We also assume that in the original schedule all available seats are utilized for all flights.

Given a schedule and delayed aircraft, we look for possible swaps between aircraft. We assume that a swap between two aircraft f_1 and f_2 is feasible if f_1 is planned to arrive at an airport at some time point T and f_2 is planned to arrive to the same airport in the time interval $[T - 60, T + 180]$ in the original schedule.

We performed all experiments on a 64-bit Windows 7 computer with 8 GB memory and Intel Core i7-3770 3.40 GHz

CPU. For each experimental setting and schedule, we generated six replications. We calculated delay costs for the delay propagation, solved the proposed nonlinear model with only cruise speed decisions (CSC), and also solved the same model with swapping as well as cruise speed decisions (S-CSC). We solved conic quadratic mixed integer reformulations described in §5 using IBM ILOG CPLEX 12.5.

Table 9 compares the performance of CSC and S-CSC with DP and gives the average CPU time for different experimental settings. The table summarizes the results for 96 problems solved by CSC and S-CSC approaches. costimp% and delayimp% give the cost ($) and delay (min) improvement achieved compared to DP cost, respectively. For ORD and DFW schedules, the average improvement in rescheduling cost achieved by CSC is 27.1% and 18.0%, respectively. The average reduction in arrival delay (in minutes) for ORD and DFW is 33.2% and 22.6%, respectively. The table shows when the delay cost w_l is higher, achievable cost improvement by CSC is higher as expected since in this case speeding up

Table 9. Rescheduling cost improvement and CPU time results.

Schedule	w_l	L_d	N_d	CSC			S-CSC		
				costimp%	delayimp%	CPU	costimp%	delayimp%	CPU
ORD	Low	Low	1	35.3	47.8	0.09	40.9	50.1	29.1
			2	23.4	34.3	0.07	25.1	35.2	45.6
		High	1	18.8	26.4	0.08	29.0	34.8	30.1
			2	15.6	22.1	0.08	22.4	26.7	55.4
	High	Low	1	43.9	47.8	0.09	47.8	51.0	25.7
			2	33.7	37.3	0.08	34.6	37.8	47.5
		High	1	24.4	27.0	0.09	40.2	51.8	36.4
			2	21.9	23.7	0.08	29.8	34.4	49.1
DFW	Low	Low	1	14.8	23.9	0.15	17.5	34.6	167.9
			2	23.5	31.7	0.17	24.8	38.5	241.5
		High	1	8.3	14.0	0.15	18.9	26.9	162.3
			2	13.6	18.8	0.14	22.2	32.5	249.1
	High	Low	1	22.0	24.5	0.16	31.1	35.4	128.3
			2	30.9	33.7	0.15	35.8	40.7	237.8
		High	1	12.5	14.3	0.16	26.6	29.9	133.0
			2	18.1	20.0	0.15	31.3	40.1	248.4

aircraft achieves more reduction in the delay cost. Table 9 also illustrates that for a given disruption(s) the CPU time requirement of calculating the optimal cruise speed is very small. For example, the average CPU time spent to solve the CSC formulation is less than 0.2 seconds for the ORD schedule. It is important to note that we limit the increase of a cruise speed by at most 10%, although it may be increased even further with the new engines. If we relax this constraint, the benefit of cruise speed control would be even higher.

In Table 9, under costimp% and delayimp% columns we give the cost and delay improvements achieved by S-CSC over DP method. The results show that considering swaps can significantly improve rescheduling costs compared to CSC approach. The average cost improvement achieved by S-CSC is 33.7% and 26.0%, respectively, for ORD and DFW schedules. For ORD and DFW the average delay improvement achieved by S-CSC is 40.2% and 34.8%, respectively. The results indicate that we are more likely to find improving swaps when the average departure delay (L_d) is longer. This observation may suggest that it is better to repair short delays by speed adjustments and consider aircraft swaps when delays are longer. In the experiments, maximum

improvement achieved by CSC and S-CSC approaches are 58.5% and 70.1%, respectively.

Moreover, the average CPU time requirement for solving the S-CSC model for ORD and DFW experiments is 40 and 196 CPU seconds, respectively. As expected, when there are two disruptions, the number of possible swaps considered in rescheduling and hence the CPU time to solve the problem increases.

In the e-companion to this paper, we give the detailed results obtained for all instances and the detailed data for each run can be obtained from the authors. Table 10 presents detailed cost results for a set of selected instances for different factor combinations used in our computational study. The DP column gives the delay cost incurred as a result of right-shifting. Under the CSC column, we give additional fuel and carbon cost incurred because of speeding up flights and delay costs. S-CSC gives additional fuel and carbon costs, delay costs, the number of aircraft swaps made, and swap related costs such as deadheading cost and spilling cost.

In problem 1, we observe that CSC approach reduces the delay costs by incurring additional fuel and carbon costs, which in turn reduces the total cost caused by the disruption. For the same instance, S-CSC finds an aircraft swap that

Table 10. Detailed cost figures for a selected set of instances.

		Factors				DP	CSC			S-CSC					
Prob	Sch	w_l	L	N_d	Rep	Delay	Fuel + Carbon	Delay	Total	Fuel + Carbon	Delay	#swap	Deadhead	Spilling	Total
1	ORD	1	1	1	1	15,090	197	11,974	12,171	147	7,359	1	1,829	51	9,386
2	ORD	0	0	1	5	3,300	361	2,019	2,381	−2,305	1,609	1	1,343	636	1,284
3	ORD	1	1	1	6	19,360	520	12,579	13,099	1,250	7,793	1	0	2,856	11,899
4	ORD	0	0	2	6	2,830	451	1,354	1,805	407	1,292	2	0	0	1,699
5	ORD	0	1	2	6	6,865	805	3,874	4,679	−2,360	4,007	2	1,111	1,416	4,173
6	ORD	1	1	2	1	22,030	353	18,350	18,703	605	14,263	1	1,086	0	15,953

could reduce the fuel, carbon, and delay costs at the same time, albeit with additional deadhead and spill costs.

Technological advancements in the airline industry have significantly reduced aircraft engine fuel consumption and emissions. As discussed in Marais and Waitz (2009), fuel consumption per passenger kilometer has decreased by 70% over the past four decades. Therefore, aircraft recovery algorithms should take into account the engine fuel efficiency while dealing with a disruption. It might be necessary to swap two aircraft just to take advantage of fuel efficiency as we observed in our computational experiments. For example in problem 2, we observe that compared to CSC, S-CSC has reduced both fuel and delay costs by performing an aircraft swap. Despite the deadhead cost incurred, the overall disruption cost is reduced. In problem 3, swapping aircraft results in spilled passenger cost but nevertheless decreases overall disruption cost for the assumed parameter values. In problem 3, compared to CSC, fuel + carbon costs are increased, but delay costs are decreased by S-CSC. In problem 4, aircraft of the same type were swapped, which resulted in zero spilling cost. Swapped aircraft pairs have the same final destination by the end of the recovery horizon, so no deadhead cost was incurred by the swaps. In problem 5, S-CSC finds two swaps to reduce disruption costs. In problem 6, a single swap found by S-CSC requires deadhead cost but still achieves a better overall solution compared to CSC. Despite an increase in fuel costs and incurred deadhead cost, the overall disruption cost is decreased by 14.7%.

Bratu and Barnhart (2006) develop an airline control center simulator to evaluate the potential impact of decisions generated by their airline recovery models. They state that recovery solutions should be generated in fewer than three minutes; otherwise, the recovery solution can become infeasible, and multiple scenario analysis cannot be conducted. The computational experiments show that this speed requirement is met to allow the proposed nonlinear optimization model to be used by operations controllers for disruption management in real time instead of relying on heuristic approaches. Moreover, solving only the proposed CSC model, which takes less than 0.17 seconds in all instances, may be sufficient to deal with short disruptions.

7.1. Nonlinear Delay Costs

As discussed earlier, almost all of the existing literature on aircraft recovery assume a linear delay cost. However, Hoffman and Ball (2000) and Hansen et al. (2001) report that the cost of a delay varies nonlinearly with the duration of the delay in practice since the delay propagates throughout the network. According to Schumer and Maloney (2008), 33.7% of delays during 2007 were attributed to late-arriving aircraft.

In order to test the use of the nonlinear delay costs, we replace the linear delay cost term in our objective function with delay costs of the form $w_l t_l^\sigma$ as discussed in §5. Table 11 summarizes the results for nonlinear delay cost with $\sigma = 1.5$ for all flights single disruptions.

With nonlinear delay costs, CSC and S-CSC models are still solved in reasonable CPU times. As expected for given experimental settings in Table 11, cost reductions achieved by CSC and S-CSC are significantly higher than the ones achieved for linear delay costs. Furthermore, the value of speed control becomes more evident for the nonlinear delay cost as a recovery strategy. To the best of our knowledge, this is the first study that considers a nonlinear delay cost for aircraft recovery.

7.2. Step Function Delay Costs

In order test the CSC and S-CSC approaches with a step delay cost function, we consider five delay intervals presented in Figure 6, (0–15, 15–30, 30–60, 60–120, 120-). For each flight l and each delay time interval m, we randomly generated c_l^m and w_l^m values. As a delay is higher, c_m^l and w_l^m are higher. We give the computational results in Table 12. The results indicate that compared to the linear delay cost case, solving the model for S-CSC with step cost functions require longer CPU times, but the instances for the schedule ORD could still be solved in 120 CPU seconds on average. Maximum CPU times observed for CSC and S-CSC approaches were 4.2 and 202.8 seconds on average, respectively.

7.3. Match-Up with Swaps

We test the match-up model for six replications resulting in 48 instances. Instead of one-day schedules used in previous

Table 11. Improvement and CPU time results for nonlinear delay costs (ORD).

Schedule	w_l	L_d	N_d	CSC costimp%	delayimp%	CPU	S-CSC costimp%	delayimp%	CPU
ORD	Low	Low	1	52.5	47.7	0.25	54.8	50.9	97.7
			2	41.8	37.0	0.26	42.4	37.6	200.1
		High	1	33.8	26.9	0.25	50.1	51.7	109.0
			2	30.0	23.7	0.28	39.3	37.9	203.9
	High	Low	1	53.8	47.8	0.25	57.0	55.2	110.8
			2	43.0	37.2	0.28	45.9	42.0	145.9
		High	1	34.7	27.0	0.31	53.7	53.0	102.5
			2	30.3	23.7	0.29	42.9	40.7	139.3

Table 12. Improvement and CPU time results for step function delay costs (ORD).

Schedule	w_l	L_d	N_d	CSC			S-CSC		
				costimp%	delayimp%	CPU	costimp%	delayimp%	CPU
ORD	Low	Low	1	38.8	46.8	3.4	44.2	49.8	91.0
			2	26.8	33.1	2.9	28.4	33.8	104.4
		High	1	22.9	26.5	2.6	34.2	39.0	105.1
			2	18.1	22.1	4.2	26.0	29.6	112.4
	High	Low	1	46.3	47.8	2.1	50.2	51.0	97.7
			2	36.3	37.3	2.1	37.3	37.7	150.1
		High	1	26.0	27.0	2.4	43.7	51.8	94.5
			2	23.3	23.7	2.9	32.9	34.4	202.8

runs, we use two-day schedules to allow more swap opportunities and to satisfy the match-up requirement. With the CSC approach we observe a 32% average improvement in disruption costs within 0.17 CPU seconds on the average. We achieve optimal solutions for 31 of the 48 instances within a time limit of 3,600 CPU seconds. For nine of these instances, the optimal solutions have improved swaps and their average solve time is 451 seconds. The average cost improvement for those instances is 36%. For the remaining 22 instances, which are solved to optimality, there is no improving swap satisfying the feasibility and/or optimality conditions. Average solution time for them is 255 CPU seconds. For the remaining 17 instances that could not be solved to optimality within 3,600 seconds, only one has an improved swap. These results indicate that the larger S-CSC model with match-up extension takes longer to solve.

8. Conclusions

There is a critical trade-off between the fuel consumption (and its adverse impact on surface air quality) and delay minimization. Although flight time controllability is a very popular local recovery strategy in practice to deal with the disruptions, its benefit has been limited because it does not consider networkwide integrated effects. To the best of our knowledge, this is the first study in which the cruise speed is included as a decision variable in an airline recovery optimization model along with the environmental constraints and cost coefficients. Flight time controllability, nonlinear delay, fuel burn, and CO_2 emission cost functions as well as binary aircraft swapping decisions complicate the aircraft recovery significantly. We utilize the recent advances in conic mixed integer programming and propose a strengthened conic formulation to mitigate the computational difficulty. Although optimization techniques are used quite extensively in the airline industry, this is the first implementation of a conic quadratic optimization approach to solve a critical aircraft recovery problem in an optimal manner. A natural extension of this study would be developing a robust airline schedule so that the disruptions can be managed in a less costly manner.

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