Low-threshold lasing eigenmodes of an infinite periodic chain of quantum wires

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We study the lasing eigenvalue problems for a periodic open optical resonator made of an infinite grating of circular dielectric cylinders standing in free space, in the E- and H-polarization modes. If possessing a "negative-absorption" refractive index, such cylinders model a chain of quantum wires made of the gain material under pumping. The initial-guess values for the lasing frequencies are provided by the plane-wave scattering problems. We demonstrate a new effect: the existence of specific grating eigenmodes that have a low threshold of lasing even if the wires are optically very thin.

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Today's technologies enable manufacturing of advanced light-emitting devices based on single or multiple quantum wires embedded in epitaxially grown semiconductor microcavities. The advantages of quantum wires with respect to quantum wells include their better thermal stability, lower chemical reactivity, and higher mechanical strength. We consider an infinite periodic chain of parallel circular quantum wires in free space as a simple model of a microcavity with a periodically structured active region. Although there are numerous publications studying transmission and reflection of plane waves by periodic grids of passive dielectric and metallic wires (for instance, see [1–3]), it looks like the associated eigenproblems have so far escaped a detailed analysis. We study such eigenproblems, for two alternative polarizations, in the modified formulation adapted to characterize the lasing [4]. By introducing the active refractive index of the wires, we obtain the possibility to determine the spectra and the associated thresholds of lasing for the eigenmodes. Here, plane-wave scattering problems serve as auxiliary ones that yield initial guesses for the lasing frequencies of eigenproblems.

The considered resonator consists of circular cylinders parallel to the z axis and periodic along the x axis; see Fig. 1. The distance between cylinder centers (i.e., period) is p, and their radii are a. We suppose that the electromagnetic field is time-harmonic $\sim \exp(-i\omega t)$ and does not vary along the z axis.

Then two alternative polarizations, E and H, can be considered separately using the E_z or $Z_0 H_z$ component of the electromagnetic field, respectively. This function must satisfy the Helmholtz equation with appropriate wavenumbers inside and outside of cylinders, the Sveshnikov radiation condition at infinity (see [5] for the explicit form), the condition of local integrability of power, and the boundary conditions demanding continuity of the tangential field components at the cylinder boundary. The free-space wavenumber is $k = \omega/c = 2\pi/\lambda$, where c is the free-space light velocity and λ is the wavelength, while inside the cylinders it is $k\nu$, where $\nu = \alpha$ in the scattering problem and $\nu = \alpha - i\gamma$ in the eigenproblem. The values of α and γ are real and positive; the former is the refractive index and the latter is the material gain that appears in the presence of a pump. For detailed discussion of the modified eigenproblem approach in the linear modeling of microcavity lasers see [4]. It has been already applied to the threshold analysis of two-dimensional entirely and partially active circular microcavities in [6,7], and to one-dimensional quantum-well equipped VCSEL-type layered cavities in [8].

Here, we use the Floquet theorem and assume that the field functions within the adjacent elementary cells of the periodic cavity are the same, i.e., $U(x+p, y) = U(x, y)$. Then the scattering problem implies normal incidence and can be reduced to the consideration of one elementary cell of the grating, similar to [1–3]. This leads to the Fredholm second-kind infinite-matrix equation, whose numerical solution has guaranteed convergence, $[I + G(\kappa; \xi, \nu)]X = B$, where we denote $\kappa = ka$ and $\xi = p/a$, I is an identity matrix, and X and B are the vector of the Floquet-harmonic amplitudes of U and the right-hand-part vector, respectively (see [1,3] for details). In the case of the eigenproblem, there is no incident field ($B = 0$), and we have to find eigenfunctions U corresponding to the eigenvalue pairs (κ, γ). This leads to solving the determinantal equation, $\det[I + G(\kappa; \xi, \nu)] = 0$,

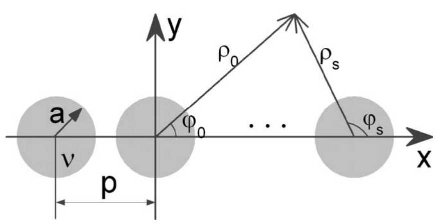

Fig. 1. Cross section of the periodic cavity of active dielectric circular cylinders or quantum wires.

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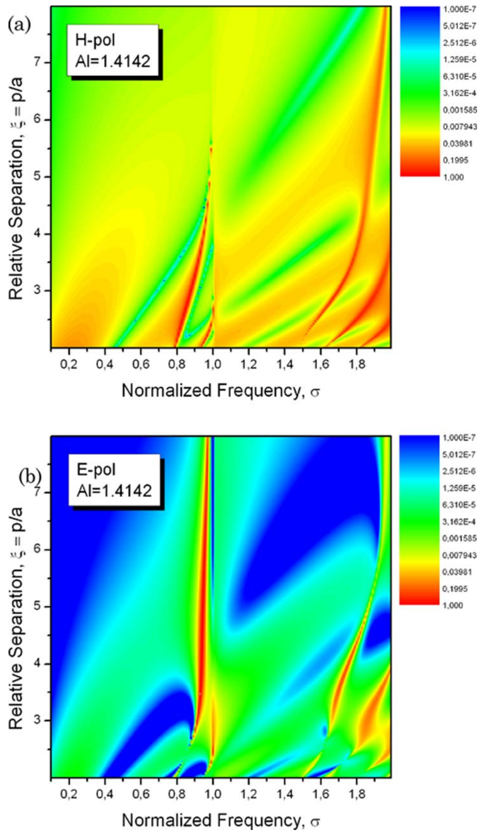

Fig. 2. (Color online) Reflectance of the circular-wire grating in the (a) H-polarization and (b) E-polarization cases as a function of the normalized frequency σ and relative distance ξ between the wires of refractive index $\alpha = 1.4142$.

where all coefficients are the same as in the scattering problem, except for ν, which is now a complex value. Note that the matrix G is similar to Eqs. (4) and (5) of [9] for a two-disk laser, with the two-term sum of Hankel functions being replaced with an *infinite lattice sum* [1–3]. Note also that similar, but still different, transcendental or determinantal equations lie in the core of numerical threshold studies of other microcavity laser configurations reported in [10,11].

In Figs. 2(a) and 2(b), we show the reflectance of the circular-wire grid for the E- and H-polarization cases, respectively, as a function of the normalized frequency, $\sigma = p/\lambda$, and relative distance between the wires whose material has refractive index $\alpha = 1.4142$. The bright ridges are the areas of intensive reflection caused by the resonances associated with the eigenmodes. Note that single-wire eigenvalues, perturbed by the presence of the other wires, exist in the case of an infinite chain as well. If $\xi \to \infty$, then the corresponding maxima of reflectance cross the vertical lines $\sigma = 1, 2, \ldots$ as inclined lines, because here $\kappa = ka \approx \text{const}$ and hence $\sigma = (\kappa/2\pi)\xi \approx \text{const}\xi$. These are *the wire eigenmodes*.

However, our main interest relates to the eigenvalues of a different nature that manifest themselves as the maxima in Figs. 2(a) and 2(b) that tend to $\sigma = 1, 2, \ldots$ if $\xi \to \infty$. Such resonances have been reported earlier in [12–14], where the plane-wave scattering by a grid of very thin dielectric wires was considered analytically. Our detailed study has revealed that they are caused by the eigenmodes of the air gap between the quantum wires and, therefore, have no counterparts among the single-wire eigenvalues. We will call them *the grating eigenmodes*, which has to be distinguished from the term *lattice modes* used as a synonym to the Floquet harmonics, which are not the eigenmodes of the grating but are the individual terms in the series field representation.

When looking for the roots of a determinantal equation, we take the frequency of the corresponding maximum of reflectance, add a small threshold of the order of 0.01, and use this data as initial guess in the iterative algorithm. In Figs. 3 and 4, we present the dependences of the lasing frequencies and thresholds

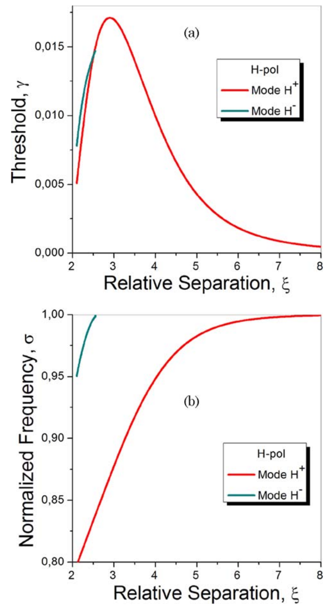

Fig. 3. (Color online) Dependences of the (a) frequencies and (b) thresholds of lasing for the H-polarized grating modes H_1^+ and H_1^- from the relative distance between the wires at $\alpha = 1.4142$.

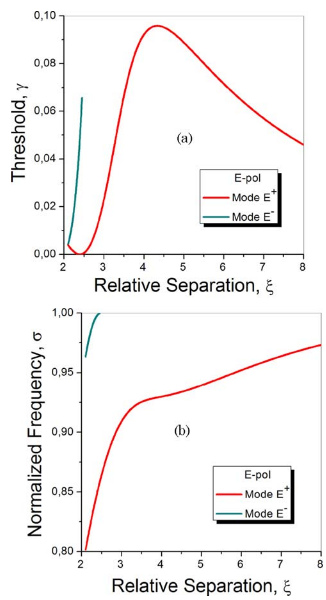

Fig. 4. (Color online) Same as in Fig. 3 for the E-polarized grating modes E_1^+ and E_1^-.

for two lowest-order E-type and H-type eigenmodes of the above-mentioned type for a grid of active quantum wires, as a function of relative distance between the wires, $\xi = p/a$. Note that the H_1^+ and E_1^+ eigenmodes have the fields symmetric across the x axis, and the eigenmodes H_1^- and E_1^- have the fields antisymmetric across that axis. The threshold material gain of the eigenmode H_1^+ reaches maximum at $\xi \approx 3.1$ and goes down if the interwire distance increases. The threshold of the eigenmode H_1^- grows monotonically; however, its frequency quickly approaches the value $\sigma = 1$. This value is the branching point for the lattice sums involved in the elements of the G matrix [1], and therefore the computations fail when the distance to it gets smaller than 10^{-7}. The lasing frequencies and thresholds of the E-type grating eigenmodes, E_1^+ and E_1^-, show similar behavior: the threshold of the former has a maximum at $\xi \approx 4.4$, and that of the latter grows monotonically while its frequency quickly approaches $\sigma = 1$. Note that the threshold gain of the H-type grating modes is much smaller than that of the E-type modes; this is apparently linked to the fact that the quantum wire material is nonmagnetic. The most interesting feature is the decrement of the thresholds for the x-even eigenmodes H_1^+ and E_1^+ if the grating of quantum wires gets sparser. This behavior is drastically different from the behavior of the lasing thresholds for the wire eigenmodes—they increase if the interwire distance gets larger and always remain finite. Moreover, the wire eigenmode thresholds (not shown here) grow if the wires get closer, in contrast to the thresholds of the grating eigenmodes presented here. Therefore, the grating eigenmodes of the infinite periodic chain of quantum wires that have subwavelength diameter are the most promising candidates for lasing when pumping is applied.

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