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Nonlinear four-terminal microstructures: A hot-spot transistor?

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Four-terminal microcontacts between metallic electrodes develop nonlinear current-voltage dependencies both in the source and control channels as well as between the channels. Theory is presented of the nonlinearity caused by the reabsorption of nonequilibrium phonons emitted in the contact by injected electrons. Temperature of the lattice due to heating by the current is of the order $T\sim eV/4$, which results in substantial increase of the resistance both in the bias direction and in the direction perpendicular to the bias. Performance characteristics of such a device at low temperature compared to the Debye temperature are quite promising for frequencies below 10^9 Hz.

I. INTRODUCTION

Point contacts between metals, and between semiconductors and semimetals, develop a number of nonlinear effects associated with the interaction between electrons and Bose-type elementary excitations (phonons, magnons, etc.). The degree of nonlinearity is generally not large (of the order d/l) in the ballistic regime in which the contact diameter (d) is much smaller than the electron and phonon mean free paths (l_e, l_{ph}); however, the nonlinear part of a current is related in a very direct way to the characteristic electron-boson interaction function $g(\omega)=\alpha^2(\omega)F(\omega)$, where $F(\omega)$ is the density of the boson excitation and $\alpha(\omega)$ the energy-dependent matrix element of the electron-boson (e.g., electron-phonon) interaction. This dependence serves as a basis of point-contact spectroscopy of elementary excitations in metals.[1-3]

The opposite regime of small electron and phonon mean free paths is less informative in that respect. However, the degree of nonlinearity in the I-V dependence increases substantially because nonequilibrium phonons emitted by "hot" electrons with the excess energy equal to the bias energy eV spend most part of their lifetime in the vicinity of constriction and therefore increase the contact resistance. The response time τ_0 for such a nonlinear conductance is quite small because of the smallness of the contact area, $\tau_0 = \max(d^2/D, \tau_{\mathrm{ph}\text{-}e})$, where D is the electron diffusion coefficient $D=\frac{1}{3}v_F l$, and $\tau_{\mathrm{ph}\text{-}e}\sim\{\omega_D^2/\lambda[T^3+(\mathrm{eV})^3]\}$ the phonon-electron relaxation time. For contact diameter $d\sim 1\ \mu$m, $T\sim 100$ K, and eV<10 mV this gives $\tau_0\sim 10^{-9}$–10^{-10} s, i.e., the operation speed of such a device may be quite large. This is supported by the observation of nonlinear behavior in metallic point contacts above this frequency range.[4]

Up-to-date technology provides a possibility of making point contacts in various configurations including multiterminal structures of micrometer and submicrometer sizes.

In this paper we present a theory of nonlinear current-voltage response of "diffusive" and/or of "thermal" point contacts (according to the definition made in Ref. 3) due to nonequilibrium phonons produced by a current. The four-terminal constricted structure allows a bias voltage in one direction, and the control of the conductance in this direction by a current in the perpendicular direction. The strong nonlinearity in both directions (channels) develops as a result of a hot spot formation at the intersection of conducting paths between the channels. The temperature of the spot is of the order of $T\sim eV/4$ and can greatly exceed the Debye temperature of a metal, and its actual temperature. In spite of the fact that the metal away from a contact remains cold, the contact resistance changes substantially and is voltage dependent.

II. THEORETICAL FORMULATION

The four-terminal structure under consideration is shown in Fig. 1. A narrow strip of metal with length L_1 and cross section S_1 carrying a current J_1 due to the connection to bulk reservoirs held at voltages V_{11} and V_{12} is in thermal and electrical contact with another strip of length L_2 and cross section S_2 connected to control reservoirs taken at voltages V_{21} and V_{22} and carrying the current J_2. We assume that the narrowest part of the four-terminal structure is thermally isolated from the substrate, i.e., its heating and cooling are solely due to electron transport and electron-phonon interaction in metal electrodes.

The equation of thermal balance is

$$-\mathrm{div}\ \mathbf{q}+\mathbf{jE}=Q, \tag{1}$$

where Q is the heat transferred between the strips. $\mathbf{q}$ and $\mathbf{j}$ are thermal flow density and electric current density, respectively, related to thermal gradient and to electric field as

$$\mathbf{q}=-\kappa\nabla T, \quad \mathbf{j}=\sigma\mathbf{E}=-\sigma\nabla\phi. \tag{2}$$

We assume that thermal conductivity κ and electrical conductivity σ are related to each other according to the Wiedemann–Franz law

$$\kappa/\sigma T=L, \tag{3}$$

where $L=(\pi^2/3)(k_B/e)^2$ is the Lorentz number (later we adopt units in which $k_B=e=1$). For the strips that are long compared with their diameters ($L_{1,2}\gg d_{1,2}$), the temperature and electrostatic potential distributions are one dimensional:

$$\begin{aligned} &T_1(x),\phi_1(x), \quad -L_1/2<x<L_1/2\\ &T_2(y),\phi_2(y), \quad -L_2/2<y<L_2/2, \end{aligned} \tag{4}$$

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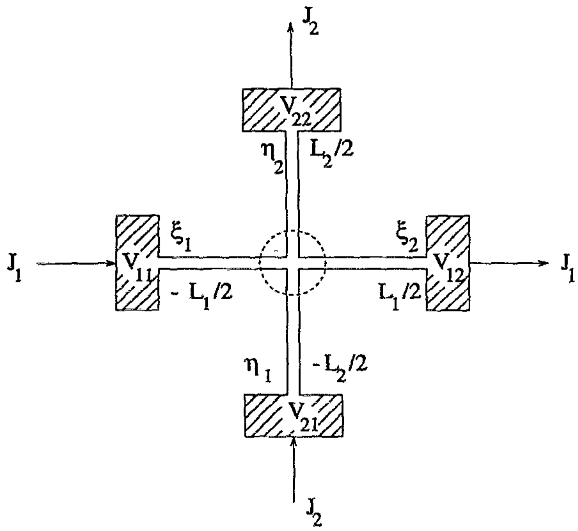

FIG. 1. Schematics of a four-terminal microstructure. The dotted line shows a hot-spot region. $V_1=V_{12}-V_{11}$, $V_2=V_{22}-V_{21}$ are voltages in the source channel (1) and in the control channel (2), respectively.

whereas heat transfer between 1 and 2 can be considered as a delta function giving

$$S_1(-\mathrm{div}\,\mathbf{q}+\mathbf{jE})_1-Q_{12}\delta(x)=0$$
$$S_2(-\mathrm{div}\,\mathbf{q}+\mathbf{jE})_2-Q_{21}\delta(y)=0, \tag{5}$$

where Q is proportional to the temperature difference between the points $x=0$ and $y=0$

$$Q_{12}=-Q_{21}=k_{12}(T_1-T_2)_0, \tag{6}$$

and the subscript "0" means $T_1(x=0)$ and $T_2(y=0)$. An estimate of heat transfer coefficient is $k_{12}\sim d_1d_2(T_1+T_2)nv_F/\epsilon_F$, where n is the electron concentration, and ϵ_F, v_F the Fermi energy and the Fermi velocity, respectively.

Let us introduce the new variables ξ, η in the x and y directions, respectively, according to

$$\sigma_1\frac{d}{dx}=\frac{d}{d\xi}, \quad \sigma_2\frac{d}{dy}=\frac{d}{d\eta}, \tag{7}$$

where $\sigma_1=\sigma[T_1(x)]$, $\sigma_2=\sigma[T_2(y)]$. $\sigma(T)$ is the temperature dependent conductivity of a metal. Equation (7) implies that

$$\sigma\delta(x)=\delta(\xi), \quad \sigma\delta(y)=\delta(\eta), \tag{8}$$

which reduces Eqs. (5) to

$$\frac{1}{2}\frac{Ld^2(T_1^2)}{d\xi^2}+\left(\frac{d\phi_1}{d\xi}\right)^2-\frac{k_{12}}{S_1}(T_1-T_2)\delta(\xi)=0$$
$$\frac{1}{2}\frac{Ld^2(T_2^2)}{d\eta^2}+\left(\frac{d\phi_2}{d\eta}\right)^2-\frac{k_{12}}{S_2}(T_1-T_2)\delta(\eta)=0. \tag{9}$$

The continuity equations $\partial j_1/\partial x=0$, $\partial j_2/\partial y=0$ give

$$\frac{d^2\phi_1}{d\xi^2}=0, \quad \frac{d^2\phi_2}{d\eta^2}=0. \tag{10}$$

The solution to Eqs. (10) is

$$\phi_1=\frac{V_{11}(\xi-\xi_2)+V_{12}(\xi_1-\xi)}{\xi_1-\xi_2}$$
$$\phi_2=\frac{V_{21}(\eta-\eta_2)+V_{22}(\eta_1-\eta)}{\eta_1-\eta_2}, \tag{11}$$

where ξ_1, ξ_2 and η_1, η_2 are values of the variables ξ, η at bulk electrodes, i.e., at $x=\mp L_1/2$ and $y=\mp L_2/2$. Potentials ϕ_1, ϕ_2 should match at points $\xi=0$, $\eta=0$ at which strips 1 and 2 are crossing each other.

The solution to Eq. (9) gives

$$T_1^2=\frac{C_1}{L_1}|\xi|+\alpha_1\xi^2+\beta_1\xi+\gamma_1$$
$$T_2^2=\frac{C_2}{L_2}|\eta|+\alpha_2\eta^2+\beta_2\eta+\gamma_2, \tag{12}$$

where

$$\alpha_1=-\frac{V_1^2}{L_1}\Big/(\xi_1-\xi_2)^2, \quad \alpha_2=-\frac{V_2^2}{L_2}\Big/(\eta_1-\eta_2)^2,$$
$$C_{1,2}=\pm\frac{k_{12}}{S_{1,2}}[T_1(0)-T_2(0)]. \tag{13}$$

The quantities $V_1=V_{11}-V_{12}$ and $V_2=V_{21}-V_{22}$ are voltages at source (1, in the x direction) and control (2, in the y direction) channels. Variables ξ_1, ξ_2, η_1, η_2 and coefficients β_1, β_2, γ_1, γ_2 are to be determined from the boundary condition

$$T_1(\xi_1)=T_b, \quad T_1(\xi_2)=T_b,$$
$$T_2(\eta_1)=T_b, \quad T_2(\eta_2)=T_b, \tag{14}$$

where T_b is the bath temperature. Then, according to the definition of variables ξ, η (7) we have

$$-\frac{L_1}{2}=\int_0^{\xi_1}\frac{d\xi}{\rho_1(\xi)}, \quad \frac{L_1}{2}=\int_0^{\xi_2}\frac{d\xi}{\rho_1(\xi)}$$
$$-\frac{L_2}{2}=\int_0^{\eta_1}\frac{d\eta}{\rho_2(\eta)}, \quad \frac{L_2}{2}=\int_0^{\eta_2}\frac{d\eta}{\rho_2(\eta)}, \tag{15}$$

where $\rho(T)=1/\sigma(T)$, thus giving another four equations for the above coefficients.

At $k_{12}=0$, the equations for T_1, T_2 are decoupled, and the solution reduces to the one found by Holm[5] and by others.[6,7] If bath temperature $T_b=0$, the temperature in the middle of a contact is related to bias voltage V according to

$$eV=\gamma T, \quad \gamma=\frac{2\pi}{\sqrt{3}}=3.62. \tag{16}$$

(Compare this with the ballistic strong phonon-trapping regime in which $eV\simeq 4$ T.[8])

In the geometry of four-terminal constriction, two films are strongly coupled and temperatures of both strips coincide at $x=0$, $y=0$. This case is achieved formally by taking the limit $k_{12}\to\infty$ in Eqs. (9). (The accuracy of this approximation is $d_{1,2}/L_{1,2}$.) Taking for simplicity $T_b=0$ and introducing a parameter

$$\lambda=(k_{12}/L)(T_1-T_2)_0 \tag{17}$$

in Eqs. (12) and (13), consider a situation in which λ remains fixed when $k_{12}\to\infty$ and $(T_1-T_2)_0\to 0$. For a symmetric configuration shown in Fig. 1, it appears that the temperature distribution is symmetric $T_1(-\xi)=T_1(\xi)$, $T_2(-\eta)=T_2(\eta)$ with

$$-\xi_1=\xi_2=\xi_0, \quad -\eta_1=\eta_2=\eta_0. \tag{18}$$

We thus obtain

$$T_1^2(\xi)=[T_0^2+(V_1^2/4L)z](1-z), \quad z=|\xi|/\xi_0$$
$$T_2^2(\eta)=[T_0^2+(V_2^2/4L)z](1-z), \quad z=|\eta|/\eta_0, \tag{19}$$

where T_0 is the temperature in the center of a contact. The latter is subject to a self-consistency relation

$$T_0^2\left(\frac{J_1V_2}{S_1}+\frac{J_2V_1}{S_2}\right)=\frac{V_1V_2}{\gamma^2}\left(\frac{J_1V_1}{S_1}+\frac{J_2V_2}{S_2}\right), \tag{20}$$

whereas currents in the longitudinal (J_1) and transverse (J_2) channels are determined as

$$J_1=\frac{S_1V_1}{L_1}\int_0^1\frac{dz}{\rho[T_1(z)]}, \quad J_2=\frac{S_2V_2}{L_2}\int_0^1\frac{dz}{\rho[T_2(z)]}. \tag{21}$$

For temperature-independent resistivity, ρ=const, Eq. (20) gives

$$T_0^2=\gamma^{-2}\,\frac{V_1^2L_2+V_2^2L_1}{L_1+L_2}, \tag{22}$$

which is an interpolation between the values corresponding to Eq. (16) for two constrictions at voltages V_1, V_2.[3]

III. NONLINEAR RESPONSE OF A CONTACT

In case of weak temperature dependence of resistivity,

$$\rho=\rho_0+\rho_{\text{ph}}(T),$$

with $\rho_{\text{ph}}\ll\rho_0$, the correction to the ohmic current at $T_b=0$ is expressed according to Eq. (21) as

$$\Delta J_1=-\frac{V_1}{L_1\rho_0^2}\times\int_0^1\rho_{\text{ph}}\left[\frac{1}{\gamma}\sqrt{\left(\frac{V_1^2L_2+V_2^2L_1}{L_1+L_2}+V_1^2z\right)(1-z)}\right]dz$$
$$\Delta J_2=-\frac{V_2}{L_2\rho_0^2}\times\int_0^1\rho_{\text{ph}}\left[\frac{1}{\gamma}\sqrt{\left(\frac{V_1^2L_2+V_2^2L_1}{L_1+L_2}+V_2^2z\right)(1-z)}\right]dz, \tag{23}$$

correspondingly in channels 1 and 2. The conductivity corrections in the channels, with $G_{1,2}=dJ_{1,2}/dV_{1,2}$, are therefore

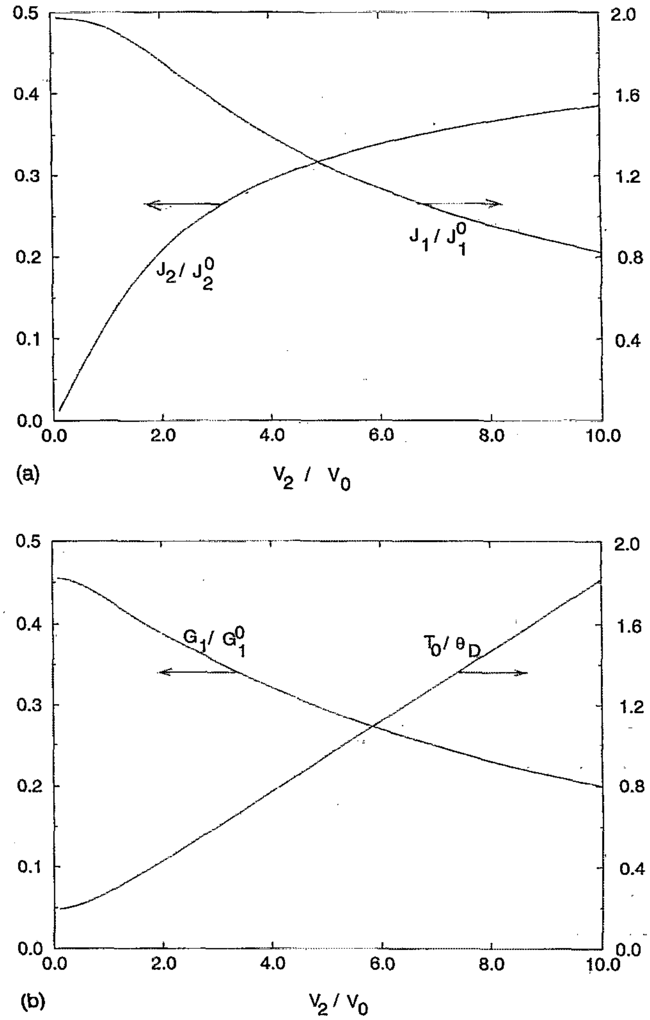

FIG. 2. (a) The dependence of the current in the longitudinal channel (J_1) and in the transverse channel (J_2) upon the voltage in the transverse (control) channel V_2 at $V_1=V_0$. (b) The hot-spot temperature T_0 and the conductance in the longitudinal channel G as a function of control channel voltage V_2. System parameters are $L_1=L_2$, $\rho_0=\rho_1$, $T_b=0$; $G_i^0=S_i/L_i\rho_0$, $J_i^0=G_i^0V_0$, $eV_0=\gamma\Theta_D$.

$$\Delta G_1(V_2)\simeq-\frac{S_1}{L_1\rho_0^2}\int_0^1\rho_{\text{ph}}\left[\frac{V_2}{\gamma}\left(\frac{L_2}{L_1+L_2}\right)^{1/2}\sqrt{1-z}\right]dz \quad \text{at } V_1\to 0$$
$$\Delta G_2(V_1)\simeq-\frac{S_2}{L_2\rho_0^2}\int_0^1\rho_{\text{ph}}\left[\frac{V_1}{\gamma}\left(\frac{L_1}{L_1+L_2}\right)^{1/2}\sqrt{1-z}\right]dz \quad \text{at } V_2\to 0. \tag{24}$$

An estimate of the cross-channel nonlinearity gives (in the case of $L_1=L_2$)

$$\left|\frac{d\ln G_2}{d\ln V_1}\right|\sim 0.2\,\frac{R_{\text{ph}}}{R_{\text{res}}}, \tag{25}$$

where R_{res} is the residual resistivity and R_{ph} the resistivity due to phonons. This implies that nonlinearity is of the order of 1 when the residual resistivity ratio (RRR) of a metal is of the order of or larger than 1.

Strong nonlinear regime (RRR$\gg$1) requires numerical solution of Eqs. (20) and (21). Doing this iteratively, we obtain the temperature of the hot-spot T_0 as a function of V_1, V_2, and the dependencies $J_1(V_1,V_2)$, $J_2(V_1,V_2)$ for currents in the corresponding channels. Typical current-voltage characteristics are shown in Fig. 2 where we have adopted the $\rho(T)$ dependence

$$\rho(T)=\rho_0+\rho_1\left[1+\left(\frac{T}{\Theta_D}\right)^5\right]^{1/5} \tag{26}$$

interpolating between the Bloch–Gruneisen law $\rho_{\text{ph}}(T)\sim T^5$ at low temperature $T\ll\Theta_D$ and the dependence $\rho_{\text{ph}}=\text{const}\,T$ at $T\gg\Theta_D$.

If the resistivity increases indefinitely with temperature, J_1 and J_2 saturate when V_1, $V_2\to\infty$ (at a constant bias in the perpendicular direction). The $T_0(V_1,V_2)$ behavior is in all cases similar to that given by Eq. (22). Note that in the case of an extremely asymmetric structure, $L_1/L_2\to 0$ or ∞, heating is determined primarily by the voltage in the longer strip.

IV. CONCLUDING REMARKS

The four-terminal structure provides a possibility of nonlinear coupling between two channels at a very small scale. Because of this, the time response is also small since the fundamental phonon-electron coupling time (at energies and temperatures of the order Θ_D) is of the order $\tau_{\text{ph-}e}\sim 10^{-10}$ s.

Nonlinear coupling appears because nonequilibrium phonons are reabsorbed in the contact, thus increasing its temperature and resistivity. The heating temperature is generally proportional to the bias voltage, Eq. (16). As at the Debye temperature, the phonon-electron mean free path is $l_{\text{ph-}e}\sim 10^{-5}$ cm, and the requirement of the small $l_{\text{ph-}e}$ to d ratio is met at the contact diameter $d\sim 1$ μm. Since the electron-phonon mean free path is of the same order at $T\sim\Theta_D$, the electron system heats up to the same temperature as the lattice does. These two systems are in equilibrium but their common temperature quite exceeds the bath temperature.

Simple nonlinear relations (21) with the universal temperature distribution (19) and maximal temperature in the middle of a contact determined self-consistently by Eq. (20) provide effective analysis of a self-induced and a cross-channel nonlinearity. The analysis carried out above is not restricted to the electron-phonon mechanism and applies to any kind of $\rho(T)$ dependence. In particular, even stronger nonlinearity is expected if the $\rho(T)$ dependence is due to the temperature driven transition between insulating and conducting states of a material, or due to a magnetic ordering. The "giant anomaly" observed in the Ni-based alloyed point contacts[6] tells us that this may be actually the case in 3D point contacts. Nevertheless, similar experiments in planar structures are lacking. Heat conductivity and thermoelectric effects in metallic point contacts have been investigated in Refs. 9 and 10. An important problem, potentially, is a heat leak from a contact to the substrate reducing the hot-spot temperature. This should be taken into consideration in the fabrication of planar four-terminal structures.

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